Cooperación Europea para la Acreditación

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Evaluación de la incertidumbre de medida en las calibraciones

OBJETO

El propósito de este documento es armonizar la evaluación de la incertidumbre de medida en EA; establecer, además de los requisitos generales de EA, los requisitos específicos sobre la expresión de la incertidumbre de medida en los certificados de calibración emitidos por los laboratorios acreditados, y ayudar a los organismos de acreditación a aplicar un enfoque coherente en la asignación de las Capacidades de Medida y Calibración a los laboratorios de calibración que acreditan. Dado que las normas que se establecen en este documento son conformes tanto a la política de la ILAC sobre la incertidumbre de calibración como a las recomendaciones de la Guía para la Expresión de la Incertidumbre de Medida, su aplicación fomentará también la aceptación mundial de los resultados de medida europeos.

Autoría
Este documento ha sido elaborado por el Comité de Laboratorios de EA.

Lengua oficial
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1 INTRODUCCIÓN

1.1. Este documento establece los principios y los requisitos para la evaluación de la incertidumbre de medida en calibraciones y para la expresión de dicha incertidumbre en los certificados de calibración, sobre la base de la política de ILAC sobre la incertidumbre en la calibración, tal como está recogida en ILAC P14* [ref.5]. Tanto ILAC-P14 como EA-4/02 son documentos obligatorios para los organismos de acreditación que sean miembros de EA. El enfoque es de carácter general, a fin de abarcar todas las áreas de calibración. El método descrito puede complementarse con recomendaciones más concretas para cada área, de manera que la información sea más fácil de aplicar. Al desarrollar estas directrices complementarias, deberán observarse los principios generales aquí expuestos, para asegurar una armonía suficiente entre las distintas áreas.

1.2. El tratamiento que se propone en este documento se corresponde con el de JCGM 100:2008, Evaluación de datos de medición - Guía para la expresión de la incertidumbre de medida (GUM 1995 con ligeras correcciones). Este documento ha sido elaborado por el Comité Conjunto para las Guías en Metrología, en el que participan BIPM, IEC, IFCC, ILAC, ISO, IUPAC, IUPAP y OIML [Ref.1]. Pero mientras que [ref. 1] establece normas generales para la evaluación y la expresión de la incertidumbre de medida que pueden aplicarse en la mayoría de los campos de mediciones físicas, este documento se centra en el método más adecuado para las mediciones realizadas por laboratorios de calibración y describe una forma armonizada y clara de evaluar y expresar la incertidumbre de medida. En todo caso, son aceptables otros enfoques propuestos por la GUM (como por ejemplo el método de Monte Carlo). Se abordan los siguientes temas:

- definiciones básicas;
- métodos para evaluar la incertidumbre de medida de las magnitudes de entrada;
- relación entre la incertidumbre de medida de la magnitud de salida y la incertidumbre de medida de las magnitudes de entrada;
- incertidumbre expandida de medida de la magnitud de salida;
- expresión de la incertidumbre de medida;
- procedimiento, paso a paso, para el cálculo de la incertidumbre de medida.

La evaluación de la incertidumbre de calibración se aborda también en varias guías sobre calibración de Euramet, disponibles en www.euramet.org

2 IDEAS GENERALES Y DEFINICIONES

Nota: En este documento, los términos que tienen una relevancia especial en el texto principal se marcan en negrita la primera vez que aparecen. El Anexo B contiene un glosario de esos términos, junto con sus referencias.

2.1. La expresión del resultado de una medición está completa sólo cuando contiene tanto el valor atribuido al mensurando como la incertidumbre de medida asociada a dicho valor. En el presente documento, todas las magnitudes que no se conocen exactamente se tratan como **variables aleatorias**, incluyendo las magnitudes de influencia que pueden afectar al valor medido.

2.2. La **incertidumbre de medida** es un parámetro no negativo, asociado al resultado de una medición, que caracteriza la dispersión de los valores que pueden atribuirse razonablemente al mensurando [ref. 4]. En el presente documento se utilizará el término abreviado **incertidumbre**, en lugar de **incertidumbre de medida**, siempre

* **Nota de ENAC**: El documento ILAC P14 está disponible en español en la página web de ENAC www.enac.es.

que no exista el riesgo de equívocos. El anexo C contiene una lista de las fuentes típicas de incertidumbre en una medición.

2.3. Los **mensurandos** son las magnitudes particulares objeto de medición. En calibración, es frecuente que sólo haya un mensurando o **magnitud de salida** Y que depende de una serie de **magnitudes de entrada** X_i (i = 1, 2 ,…, N), de acuerdo con la relación funcional

$$Y = f(X_1, X_2, \ldots, X_N) \qquad (2.1)$$

La función modelo *f* representa el procedimiento de medida y el método de evaluación. Describe cómo se obtienen los valores de la magnitud de salida *Y* a partir de los valores de las magnitudes de entrada X_i. En la mayoría de los casos, la función modelo corresponde a una sola expresión analítica, pero en otros casos se necesitan varias expresiones de este tipo que incluyan correcciones y factores de corrección para los efectos sistemáticos, en cuyo caso existe una relación más complicada que no se expresa explícitamente como una función. Es más, *f* puede determinarse experimentalmente, existir solo como un algoritmo de cálculo que deba evaluarse numéricamente, o ser una combinación de todo ello.

2.4. El conjunto de magnitudes de entrada X_i puede agruparse en dos categorías, según la forma en que se haya determinado el valor de la magnitud y la incertidumbre asociada al mismo:

(a) magnitudes cuyo valor estimado y cuya incertidumbre asociada se determinan directamente en la medición. Estos valores pueden obtenerse, por ejemplo, a partir de una única observación, de observaciones reiteradas o juicios basados en la experiencia. Pueden requerir la determinación de correcciones en las lecturas de los instrumentos o de las magnitudes de influencia, como la temperatura ambiental, la presión barométrica o la humedad relativa;

(b) magnitudes cuyo valor estimado e incertidumbre asociada se incorporan a la medición desde fuentes externas, tales como magnitudes asociadas a patrones de medida calibrados, materiales de referencia certificados o datos de referencia obtenidos de manuales.

2.5. Una estimación del mensurando *Y*, o **estimación de salida**, expresada por *y*, se obtiene de la ecuación (2.1) utilizando las **estimaciones de entrada** x_i como valores de las magnitudes de entrada X_i

$$y = f(x_1, x_2, \ldots, x_N) \qquad (2.2)$$

Se supone que los valores de entrada son estimaciones óptimas en las que se han corregido todos los efectos significativos para el modelo. De lo contrario, se habrán introducido las correcciones necesarias como magnitudes de entrada diferentes.

2.6. En el caso de las variables aleatorias, se utiliza como medida de la dispersión de los valores, la **varianza** de su distribución o la raíz cuadrada positiva de la varianza, denominada **desviación típica**. La **incertidumbre típica de medida** asociada a la estimación de salida o resultado de medida y, expresada por $u(y)$, es la desviación típica del mensurando *Y*. Se determina a partir de los valores estimados x_i de las magnitudes de entrada X_i y sus incertidumbres típicas asociadas $u(x_i)$. La incertidumbre típica asociada a un valor estimado tiene la misma dimensión que éste. En algunos casos puede utilizarse la **incertidumbre típica relativa de medida**, que es la incertidumbre típica de medida asociada a un valor estimado dividida por el módulo de dicho valor estimado y, por consiguiente, es adimensional. Este concepto no es aplicable cuando el valor estimado es cero.

3 EVALUACIÓN DE LA INCERTIDUMBRE DE MEDIDA DE LAS ESTIMACIONES DE ENTRADA

3.1 Consideraciones generales

3.1.1 La incertidumbre de medida asociada a las estimaciones de entrada se evalúa utilizando un método 'tipo A' o un método 'tipo B'. La **evaluación tipo A de la incertidumbre típica** se realiza mediante el análisis estadístico de una serie de observaciones. En este caso, la incertidumbre típica es la desviación típica experimental de la media resultante de un procedimiento de promediado o de un análisis de regresión adecuado. La **evaluación tipo B de la incertidumbre típica** se realiza por un procedimiento distinto al análisis estadístico de una serie de observaciones. En este caso, la evaluación de la incertidumbre típica se basa en otros conocimientos científicos.

Nota: En algunas ocasiones, poco frecuentes en calibración, todos los valores posibles de una magnitud caen a un mismo lado de un único valor límite. Un caso bien conocido es el llamado "error de coseno". Para el tratamiento de estos casos especiales, véase [ref. 1].

3.2 Evaluación tipo A de la incertidumbre típica

3.2.1 La evaluación tipo A de la incertidumbre típica se utiliza cuando se han realizado varias observaciones independientes de una de las magnitudes de entrada bajo las mismas condiciones de medida. Si el proceso de medida tiene suficiente resolución, se podrá observar una dispersión de los valores obtenidos.

3.2.2 Supóngase que la magnitud de entrada X_i, medida repetidas veces es la magnitud Q. Con n ($n > 1$) observaciones estadísticamente independientes, el valor estimado de la magnitud Q es $\overline{q}$, la **media aritmética** o **promedio** de todos los valores observados q_j (j = 1, 2, …, n)

$$\overline{q} = \frac{1}{n}\sum_{j=1}^{n} q_j \qquad (3.1)$$

La incertidumbre de medida asociada al valor estimado $\overline{q}$ se evalúa de acuerdo con uno de los métodos siguientes:

(a) Una estimación de la varianza de la distribución de probabilidad subyacente es la **varianza experimental** $s^2(q)$ de los valores q_j dada por

$$s^2(q) = \frac{1}{n-1}\sum_{j=1}^{n} (q_j - \overline{q})^2 \qquad (3.2)$$

Su raíz cuadrada (positiva) se denomina **desviación típica experimental**. La mejor estimación de la varianza de la media aritmética $\overline{q}$ es la **varianza experimental de la media (aritmética)**, que viene dada por

$$s^2(\overline{q}) = \frac{s^2(q)}{n} \tag{3.3}$$

Su raíz cuadrada (positiva) se denomina **desviación típica experimental de la media (aritmética).** La incertidumbre típica $u(\overline{q})$ asociada a la estimación de entrada $\overline{q}$ es la desviación típica experimental de la media

$$u(\overline{q}) = s(\overline{q}) \tag{3.4}$$

Advertencia: Generalmente, cuando el número n de mediciones repetidas es pequeño ($n < 10$), la evaluación Tipo A de la incertidumbre típica, expresada por la ecuación (3.4), puede no ser fiable. Si no se puede aumentar el número de observaciones, tendrán que considerarse otros métodos descritos en el texto para evaluar la incertidumbre típica.

(b) Cuando una medición está bien caracterizada y bajo control estadístico, es posible disponer de una **estimación combinada o acumulada de la varianza** s_{P}^2, que caracteriza la dispersión mejor que la desviación típica estimada a partir de un número limitado de observaciones. Si, en ese caso, el valor de la magnitud de entrada Q se calcula como la media aritmética $\overline{q}$ de un pequeño número n de observaciones independientes, la varianza de la media aritmética puede estimarse como

$$s^2(\overline{q}) = \frac{s_{\mathrm{P}}^2}{n} \tag{3.5}$$

La incertidumbre típica se deduce de este valor utilizando la ecuación (3.4).

3.3 Evaluación tipo B de la incertidumbre típica

3.3.1 La evaluación tipo B de la incertidumbre típica asociada a un valor estimado x_i de una magnitud de entrada X_i se realiza por medios distintos al análisis estadístico de una serie de observaciones. La incertidumbre típica $u(x_i)$ se evalúa aplicando un juicio científico basado en toda la información disponible sobre la posible variabilidad de X_i. Los valores en esta categoría pueden obtenerse a partir de

- datos de mediciones anteriores;
- experiencia o conocimientos generales sobre el comportamiento y las propiedades de los materiales e instrumentos relevantes;
- especificaciones del fabricante;
- datos incluidos en certificados de calibración y en otros certificados;
- incertidumbres asignadas a datos de referencia tomados de manuales.

3.3.2 El uso adecuado de la información disponible para una evaluación tipo B de la incertidumbre típica de medida exige entendimiento, basado en los conocimientos generales y en la experiencia. Es una destreza que puede adquirirse con la práctica. Una evaluación tipo B de la incertidumbre típica que tenga una base sólida puede ser tan fiable como una evaluación tipo A, especialmente cuando esta última se basa solo en un número comparativamente pequeño de observaciones estadísticamente independientes. Deben distinguirse los casos siguientes:

(a) Cuando sólo se conoce **un valor único** de la magnitud X_i, por ejemplo, el valor de una única medición, el valor resultante de una medición previa, un valor de referencia obtenido de la literatura o el valor de una corrección, se utilizará ese valor como x_i. Se adoptará, siempre que se disponga de ella, la incertidumbre típica $u(x_i)$ asociada a x_i. En caso contrario, deberá calcularse a partir de datos inequívocos sobre la incertidumbre. Si no puede aumentarse el número de observaciones, ha de considerarse un enfoque diferente de estimación de la incertidumbre típica, recogido en el punto b).

(b) Cuando se pueda suponer una **distribución de probabilidad** para la magnitud X_i, basada en la teoría o en la experiencia, la esperanza matemática o valor esperado y la raíz cuadrada de la varianza de esa distribución, deben tomarse como la estimación x_i y la incertidumbre típica asociada $u(x_i)$, respectivamente.

(c) Si sólo pueden estimarse unos límites **superior** e **inferior** a_+ y a_- para el valor de la magnitud X_i (por ejemplo, especificaciones del fabricante de un instrumento de medida, un intervalo de temperaturas, un error de redondeo o de truncamiento resultante de la reducción automatizada de los datos), hay que suponer una distribución de probabilidad con una densidad de probabilidad constante entre dichos límites (distribución de probabilidad rectangular) para la posible variabilidad de la magnitud de entrada X_i. Según el caso (b) anterior, se obtiene

$$x_i = \frac{1}{2}(a_+ + a_-) \tag{3.6}$$

para el valor estimado y

$$u^2(x_i) = \frac{1}{12}(a_+ - a_-)^2 \tag{3.7}$$

para el cuadrado de la incertidumbre típica. Si la diferencia entre los límites se expresa como 2a, la ecuación (3.7) se convierte en

$$u^2(x_i) = \frac{1}{3}a^2 \tag{3.8}$$

La distribución rectangular es una descripción razonable en términos de probabilidad del conocimiento que se tiene sobre la magnitud de entrada X_i cuando no existe ninguna otra información más que sus límites de variabilidad. Pero si se sabe que los valores de la magnitud en cuestión próximos al centro del intervalo de variabilidad son más probables que los valores próximos a los extremos, un modelo más adecuado sería una distribución triangular o normal. A la inversa, si son más probables los valores cercanos a los extremos, puede ser más adecuada una distribución en U. Para la evaluación de la incertidumbre en estos casos, véase [ref.1]

4 CÁLCULO DE LA INCERTIDUMBRE TÍPICA DE LA ESTIMACIÓN DE SALIDA

4.1 Para las magnitudes de entrada no correlacionadas, el cuadrado de la incertidumbre típica asociada a la estimación de salida y viene dado por

$$u^2(y) = \sum_{i=1}^{N} u_i^2(y) \tag{4.1}$$

Nota: Hay casos, poco frecuentes en calibración, en los que la función modelo es claramente no lineal o en que algunos de los coeficientes de sensibilidad [véanse las ecuaciones (4.2) y (4.3)] se anulan y tienen que incluirse términos de orden superior en la ecuación (4.1). Para el tratamiento de estos casos especiales, véase ref. 1.

La magnitud $u_i(y)$ (i = 1, 2, …, N) es la contribución a la incertidumbre típica asociada a la estimación de salida y resultante de la incertidumbre típica asociada a la estimación de entrada x_i

$$u_i(y) = c_i u(x_i) \tag{4.2}$$

donde c_i es el **coeficiente de sensibilidad** asociado a la estimación de entrada x_i, es decir, la derivada parcial de la función modelo f con respecto a X_i, evaluada para las estimaciones de entrada x_i,

$$c_i = \frac{\partial f}{\partial x_i} = \left.\frac{\partial f}{\partial X_i}\right|_{X_1 = x_1 \ldots X_N = x_N} \tag{4.3}$$

4.2 El coeficiente de sensibilidad c_i describe el grado en que la que la estimación de salida y está influida por las variaciones en la estimación de entrada x_i. Puede evaluarse a partir de la función modelo f según la ecuación (4.3) o utilizando métodos numéricos, es decir, calculando la variación en la estimación de salida y como consecuencia de una variación correspondiente en la estimación de entrada x_i de $+u(x_i)$ y $-u(x_i)$ y tomando como valor de c_i la diferencia resultante en y dividida por $2u(x_i)$. En algunas ocasiones puede ser más adecuado determinar con un experimento la variación en la estimación de salida y repitiendo la medición, por ejemplo, en $x_i \pm u(x_i)$.

4.3 Aunque $u(x_i)$ es siempre positiva, la contribución $u_i(y)$ según la ecuación (4.2) puede ser positiva o negativa, dependiendo del signo del coeficiente de sensibilidad c_i. El signo de $u_i(y)$ tiene que tenerse en cuenta en el caso de magnitudes de entrada correlacionadas, véase la ecuación (D4) del Anexo D.

4.4 Si la función modelo f es una suma o diferencia de las magnitudes de entrada X_i

$$f(X_1, X_2, \ldots, X_N) = \sum_{i=1}^{N} p_i X_i \tag{4.4}$$

la estimación de salida según la ecuación (2.2) viene dada por la correspondiente suma o diferencia de las estimaciones de entrada

$$y = \sum_{i=1}^{N} p_i x_i \tag{4.5}$$

mientras que los coeficientes de sensibilidad son p_i y la ecuación (4.1) se convierte en

$$u^2(y) = \sum_{i=1}^{N} p_i^2 u^2(x_i) \tag{4.6}$$

4.5 Si la función modelo *f* es un producto o cociente de las magnitudes de entrada X_i

$$f(X_1, X_2, \ldots, X_N) = c \prod_{i=1}^{N} X_i^{p_i} \tag{4.7}$$

la estimación de salida es de nuevo el correspondiente producto o cociente de las estimaciones de entrada

$$y = c \prod_{i=1}^{N} x_i^{p_i} \tag{4.8}$$

En este caso, los coeficientes de sensibilidad son $p_i y/x_i$ y de la ecuación (4.1) se obtiene una expresión análoga a la ecuación (4.6), si se utilizan incertidumbres típicas relativas $w(y) = u(y)/|y|$ y $w(x_i) = u(x_i)/|x_i|$,

$$w^2(y) = \sum_{i=1}^{N} p_i^2 w^2(x_i) \tag{4.9}$$

4.6 Si dos magnitudes de entrada X_i y X_k están **correlacionadas** en cierto grado, es decir, si son mútuamente dependientes de una forma u otra, su **covarianza** también tiene que considerarse como contribución a la incertidumbre. En el Anexo D se explica el modo de proceder. La capacidad de tener en cuenta el efecto de las correlaciones depende del conocimiento que se tenga del proceso de medición y del juicio sobre las dependencias mutuas de las magnitudes de entrada. En general, debe tenerse en cuenta que despreciar las correlaciones entre las magnitudes de entrada puede conducir a una estimación incorrecta de la incertidumbre típica del mensurando.

4.7 La covarianza asociada a los valores estimados de dos magnitudes de entrada X_i y X_k puede considerarse igual a cero o insignificante si

(a) las magnitudes de entrada X_i y X_k son independientes, debido, por ejemplo, a que han sido observadas reiteradamente pero no simultáneamente en diferentes experimentos independientes o a que representan magnitudes resultantes de diferentes evaluaciones realizadas independientemente, o si

(b) cualquiera de las magnitudes de entrada X_i y X_k puede tratarse como constante, o si

(c) no existe ninguna información que indique la existencia de una correlación entre las magnitudes de entrada X_i y X_k.

En algunas ocasiones, las correlaciones pueden eliminarse mediante la elección adecuada de la función modelo.

4.8 El análisis de la incertidumbre de una medición –a veces llamado balance de incertidumbre– debe incluir una lista de todas las fuentes de incertidumbre, junto con las incertidumbres típicas de medida asociadas y los métodos para evaluarlas. En el caso de mediciones repetidas, debe indicarse también el número n de observaciones. Para mayor claridad, se recomienda presentar los datos pertinentes para ese análisis en forma de tabla. En esa tabla, las magnitudes deben expresarse mediante un símbolo físico X_i o un identificador breve. Para cada una de ellas debe indicarse, al menos, el valor estimado x_i, la incertidumbre típica de medida asociada $u(x_i)$, el coeficiente de sensibilidad c_i y las diferentes contribuciones a la incertidumbre $u_i(y)$. Junto con los valores numéricos de la tabla debe indicarse la unidad de medida de cada magnitud.

4.9 En la tabla 4.1 se ofrece un ejemplo de ese formato, aplicable al caso de magnitudes de entrada no correlacionadas. La incertidumbre típica asociada al resultado de medida $u(y)$ que aparece en la esquina inferior derecha de la tabla es la raíz cuadrada de la suma de todas las contribuciones a la incertidumbre que aparecen en la última columna de la derecha. No hay que cumplimentar la parte gris de la tabla.

Tabla 4.1: Tabla esquemática para la presentación ordenada de las magnitudes, estimaciones, incertidumbres típicas, coeficientes de sensibilidad y contribuciones a la incertidumbres utilizadas en el análisis de la incertidumbre de una medición.

Magnitud X_i	Estimación x_i	Incertidumbre típica $u(x_i)$	Distribución de probabilidad	Coeficiente de sensibilidad c_i	Contribución a la incertidumbre típica $u_i(y)$
X_1	x_1	$u(x_1)$		c_1	$u_1(y)$
X_2	x_2	$u(x_2)$		c_2	$u_2(y)$
:	:	:		:	:
X_N	x_N	$u(x_N)$		c_N	$u_N(y)$
Y	y				$u(y)$

5 INCERTIDUMBRE EXPANDIDA DE MEDIDA

5.1 Se ha decidido en EA que los laboratorios de calibración acreditados por sus miembros declaren una **incertidumbre expandida de medida** U, obtenida multiplicando la incertidumbre típica $u(y)$ de la estimación de salida y por un **factor de cobertura** k,

$$U = ku(y) \qquad (5.1)$$

Cuando se puede atribuir una distribución normal (gausiana) al mensurando y la incertidumbre típica asociada a la estimación de salida tiene suficiente fiabilidad, debe utilizarse el factor de cobertura usual $k = 2$. La incertidumbre expandida asociada corresponde a una **probabilidad de cobertura** de, aproximadamente, un 95%. Estas condiciones se cumplen en la mayoría de los casos en calibración.

5.2 La hipótesis de una distribución normal no siempre puede confirmarse experimentalmente con facilidad. Sin embargo, cuando varias componentes de la incertidumbre (es decir, $N \geq 3$), derivadas de distribuciones de probabilidad de magnitudes independientes con buen con buen comportamiento, por ejemplo distribuciones normales o distribuciones rectangulares, contribuyen de manera comparable a la incertidumbre típica asociada a la estimación de salida, se cumplen las condiciones del teorema central del límite y se puede suponer, con un elevado grado de aproximación, que la distribución de la magnitud de salida es normal.

5.3 La fiabilidad de la incertidumbre típica asociada a la estimación de salida se determina por sus grados efectivos de libertad (véase Anexo E). En todo caso, se cumple el criterio de fiabilidad siempre que ninguna de las contribuciones a la incertidumbre se obtenga empleando una evaluación tipo A basada en menos de diez observaciones repetidas.

5.4 Si no se cumple alguna de estas condiciones (normalidad o fiabilidad suficiente), el factor de cobertura usual $k = 2$ puede producir una incertidumbre expandida correspondiente a una probabilidad de cobertura inferior al 95%. En estos casos, para garantizar que el valor de la incertidumbre expandida se corresponde con la misma probabilidad de cobertura que en el caso normal, tienen que utilizarse otros procedimientos. La utilización de aproximadamente la misma probabilidad de cobertura es esencial para comparar los resultados de dos mediciones de la misma magnitud, como sucede cuando hay que evaluar los resultados de una intercomparación o hay que verificar el cumplimiento de unas especificaciones.

5.5 Incluso cuando pueda suponerse una distribución normal, puede ocurrir que la incertidumbre típica asociada a la estimación de salida no tenga la suficiente fiabilidad. En este caso, si no es eficiente aumentar el número n de mediciones repetidas ni utilizar una evaluación tipo B en lugar de una evaluación tipo A poco fiable, debe utilizarse el método que se describe en el Anexo E.

5.6 En el resto de los casos, es decir, en todos aquellos en los que no puede justificarse la hipótesis de una distribución normal, debe utilizarse información sobre la distribución de probabilidad real de la estimación de salida para obtener un valor del factor de cobertura k que corresponda a una probabilidad de cobertura de aproximadamente el 95%.

6 PROCEDIMIENTO, PASO A PASO, PARA EL CÁLCULO DE LA INCERTIDUMBRE DE MEDIDA

6.1 A continuación se ofrece una guía para la aplicación práctica de este documento (véanse los ejemplos del suplemento):

(a) Expresar en términos matemáticos la dependencia del mensurando (magnitud de salida) Y respecto de las magnitudes de entrada X_i según la ecuación (2.1). En caso de comparación directa de dos patrones, la ecuación puede ser muy simple, por ejemplo, $Y = X_1+X_2$.

(b) Identificar y aplicar todas las correcciones significativas.

(c) Hacer una lista de todas las fuentes de incertidumbre en forma de análisis de la incertidumbre según el apartado 4.

(d) Calcular la incertidumbre típica $u(\overline{q})$ de magnitudes medidas reiteradamente conforme a la subsección 3.2.

(e) Para valores únicos, como los resultantes de mediciones previas, valores de corrección o valores tomados de la literatura técnica, adoptar la incertidumbre típica cuando se dé o pueda calcularse según el párrafo 3.3.2 (a). Prestar atención a la representación de la incertidumbre utilizada. Si no se dispone de datos de los cuales pueda derivarse la incertidumbre típica, declarar un valor de $u(x_i)$ basándoose en la experiencia científica.

(f) Para las magnitudes de entrada cuya distribución de probabilidad se conozca o pueda suponerse, calcular la esperanza matemática y la incertidumbre típica $u(x_i)$ conforme al párrafo 3.3.2 (b). Si sólo se dan o pueden estimarse los límites superior e inferior, calcular la incertidumbre típica $u(x_i)$ con arreglo al párrafo 3.3.2 (c).

(g) Calcular para cada magnitud de entrada X_i la contribución $u_i(y)$ a la incertidumbre asociada a la estimación de salida resultante de la estimación de entrada x_i según las ecuaciones (4.2) y (4.3) y sumar los cuadrados tal como se describe en la ecuación (4.1) para obtener el cuadrado de la incertidumbre típica $u(y)$ del mensurando. Si se sabe que las magnitudes de entrada están correlacionadas, aplicar el procedimiento que se describe en el Anexo D.

(h) Calcular la incertidumbre expandida U multiplicando la incertidumbre típica $u(y)$ asociada a la estimación de salida por un factor de cobertura k elegido conforme al apartado 5.

(i) Informar del resultado de la medición incluyendo el valor estimado y del mensurando, la incertidumbre expandida asociada U y el factor de cobertura k en el certificado de calibración con arreglo a lo indicado en la sección 6 de ILAC P14 [5] y de ILAC P15 8[5].

7 REFERENCIAS

[1] JCGM 100:2008 GUM 1995 con ligeras correcciones, Evaluación de datos de medición - Guía para la expresión de la incertidumbre de medida. (Disponible en www.bipm.org)

[2] ISO/IEC 17025/2005, Requisitos generales para la competencia de los laboratorios de ensayo y calibración.

[3] JCGM 200:2008 Vocabulario internacional de metrología - Conceptos fundamentales y básicos y términos asociados (Disponible en www.BIPM.org)

[4] ISO 3534-1, Estadísticas. Vocabulario y símbolos-Parte 1: Términos estadísticos generales y términos empleados en el cálculo de probabilidades - (ISO 3534-1:2006)

[5] ILAC P14:12/2010*- Política de la ILAC sobre la incertidumbre en la calibración

[6] JCGM 104:2009 Evaluación de datos de medición - Introducción a la “Guía para la expresión de la incertidumbre de medida” y documentos relacionados. (disponible en www.bipm.org)

* **Nota de ENAC**: El documento ILAC P14 está disponible en español en la página web de ENAC www.enac.es.

ANEXO A Capacidad de medida y calibración

El concepto de capacidad de medida y calibración, CMC, se trata en profundidad en el documento sobre las capacidades de media y calibración adoptado por el Grupo de trabajo conjunto BIPM/ILAC el 7 de septiembre de 2007. Este documento se incluye como anexo en la Política de la ILAC sobre la incertidumbre en las calibraciones, que constituye la base de un enfoque armonizado sobre la CMC entre los laboratorios acreditados de todo el mundo [ref.5].

Cuando los laboratorios acreditados establezcan su CCM, utilizarán los métodos para la evaluación de la incertidumbre descritos en este documento.

ANEXO B Glosario de algunos términos utilizados

B.1 **media aritmética** ([ref.1] sección C.2.19)
valor medio; suma de valores dividida por el número de valores

B.2 **capacidad de medida y calibración**

La capacidad de medida y calibración (CCM) se expresa en términos de:

1. Mensurando o material de referencia;
2. Método/procedimiento de calibración/medida y/o tipo de instrumento/material que ha de calibrarse/medirse;
3. Intervalo de medida y otros parámetros, cuando proceda, por ejemplo, la frecuencia de la tensión aplicada;
4. Incertidumbre de medida.

Para una explicación completa, véase ref.5.

B.3 **coeficiente de correlación** ([ref. 1] sección C.3.6)
Medida de la dependencia relativa mutua de dos variables, igual al cociente entre su covarianza y la raíz cuadrada positiva del producto de sus varianzas. Para una descripción más elaborada, véase ref.1.

B.4 **covarianza** ([ref. 1] sección C.3.4)
Medida de la dependencia mutua de dos variables aleatorias, igual al valor esperado del producto de las desviaciones de las dos variables aleatorias respecto de sus respectivos valores esperados. Puede encontrarse la definición completa en ref.1.

B.5 **factor de cobertura** ([ref. 3] término 2.3.8)
Número mayor que uno por el que se multiplica una incertidumbre de medida típica combinada para obtener una incertidumbre de medida expandida.

B.6 **probabilidad de cobertura** [ref. 3] término 2.3.7,
Probabilidad de que el conjunto de los valores verdaderos de un mesurando esté contenido en un intervalo de cobertura determinado. Nota: El término “valor verdadero” (ver Anexo D) no se utiliza en esa Guía por las razones expuestas en D.3.5; los términos “valor de un mensurando” (o de una magnitud) y “valor verdadero de un mensurando” (o de una magnitud) se consideran equivalentes. (GUM 3.1.1) Véase también ref.6 (JCGM 104:2009) capítulo 1.

B.7 **desviación típica experimental** ([ref. 1] sección 4.2.2)
La raíz cuadrada positiva de la varianza experimental.

B.8 **incertidumbre expandida (de medida)** ([ref. 3] término 2.3.5)
Producto de la incertidumbre de medida típica combinada y un factor mayor que uno.

B.9 **varianza experimental** ([referencia. 1] sección 4.2.2)
Magnitud que caracteriza la dispersión de los resultados de una serie de n observaciones del mismo mensurando dada por la ecuación (3.2) del texto.

B.10 **estimación de entrada** ([ref. 1] sección 4.1.4 y C2.26)
Valor estimado de una magnitud de entrada utilizado en la evaluación del resultado de una medición.

B.11 **magnitud de entrada** ([ref. 1] sección 4.1.2)
Magnitud de la que depende el mensurando y que se tiene en cuenta en el proceso de evaluación del resultado de una medición.

B.12 **mensurando** ([ref. 3] término 2.3). Magnitud que se pretende medir

B.13 **incertidumbre de medida**, incertidumbre ([ref.3] sección 2.26)
Parámetro no negativo que caracteriza la dispersión de los valores de la magnitud que se atribuyen a un mensurando, basado en la información utilizada.

B.14 **estimación de salida** ([ref. 1] sección 4.1. y C2.26)
Resultado de una medición calculado, a partir de las estimaciones de entrada, por la función modelo.

B.15 **magnitud de salida** ([ref. 1] sección 4.1.2)
Magnitud que representa al mensurando en la evaluación de un resultado de medida.

B.16 **estimación combinada de la varianza** ([ref. 1] sección 4.2.4)
Valor estimado de la varianza experimental obtenido de una larga serie de observaciones del mismo mensurando en mediciones bien caracterizadas y bajo control estadístico.

B.17 **distribución de probabilidad** ([ref. 1] sección C.2.3)
Función que da la probabilidad de que una variable aleatoria adopte un valor dado cualquiera o pertenezca a un determinado conjunto de valores

B.18 **variable aleatoria** ([ref. 1] sección C.2.2)
Variable que puede adoptar cualquiera de los valores de un conjunto determinado de valores y a la que se asocia una distribución de probabilidad.

B.19 **incertidumbre típica relativa de medida** ([ref. 3] sección 2.3.2)
Incertidumbre típica de medida dividida por el valor absoluto del valor medido.

B.20 **coeficiente de sensibilidad asociado a una estimación de entrada** ([ref. 1] sección 5.1.3)
Variación diferencial en la estimación de salida generada por una variación diferencial en una estimación de entrada dividida por la variación en esa estimación de entrada.

B.21 **desviación típica** ([ref. 1] sección C.2.12)
Raíz cuadrada positiva de la varianza.

B.22 **incertidumbre típica de medida** ([ref. 3] término 2.3.0)
Incertidumbre de medida expresada como desviación típica.

B.23 **Evaluación tipo A de la incertidumbre de medida** ([ref. 3] sección 2.28)
Estimación de una componente de la incertidumbre de medida por análisis estadístico de los valores medidos obtenidos en condiciones de medida definidas.

B.24 **Evaluación tipo B de la incertidumbre de medida** ([ref. 3] término 2.29)
Estimación de una componente de la incertidumbre de medida obtenida de manera distinta a una evaluación tipo A.

B.25 **balance de incertidumbre** ([ref. 3] sección 2.33)
Declaración de una incertidumbre de medida, de los componentes de esa incertidumbre de medida y de su cálculo y combinación.

B.26 **varianza** ([ref. 1] sección C.2.11)
Esperanza matemática del cuadrado de la variable aleatoria centrada.

ANEXO C Fuentes de incertidumbre de medida

C1 La incertidumbre del resultado de una medición refleja la falta de un conocimiento completo del valor del mensurando. Un conocimiento completo exigiría una cantidad infinita de información. El conjunto de fenómenos que contribuyen a la incertidumbre y, por tanto, al hecho de que el resultado de una medición no pueda caracterizarse por un único valor se denomina fuentes de incertidumbre. En la práctica, hay muchas posibles fuentes de incertidumbre en una medición [ref. 1], entre ellas las siguientes:

(a) definición incompleta del mensurando;

(b) realización imperfecta de la definición del mensurando;

(c) muestreo no representativo: la muestra medida no representa el mensurando definido;

(d) conocimiento inadecuado de los efectos de las condiciones ambientales o mediciones imperfectas de las mismas;

(e) sesgo personal en la lectura de instrumentos analógicos;

(f) resolución finita del instrumento o umbral de discriminación finito;

(g) valores inexactos de los patrones de medida y de los materiales de referencia;

(h) valores inexactos de las constantes y de otros parámetros obtenidos de fuentes externas y utilizados en el algoritmo para la reducción de datos;

(i) aproximaciones e hipótesis incorporadas en el método y el procedimiento de medida;

(j) variaciones en observaciones repetidas del mensurando realizadas en condiciones aparentemente idénticas.

C2 Estas fuentes no son necesariamente independientes. Algunas de las fuentes (a) a (i) pueden contribuir a (j).

ANEXO D Magnitudes de entrada correlacionadas

D1 Si se sabe que dos magnitudes de entrada X_i y X_k están correlacionadas en cierto grado –es decir, si dependen la una de la otra de alguna manera– la **covarianza** asociada a los dos valores estimados x_i y x_k

$$u(x_i, x_k) = u(x_i)u(x_k)r(x_i, x_k) \qquad (i \neq k) \tag{D.1}$$

tiene que considerarse como una contribución adicional a la incertidumbre. El grado de correlación se caracteriza por el **coeficiente de correlación** $r(x_i, x_k)$ (donde $i \neq k$ y $|r| \leq 1$).

D2 En el caso de n parejas independientes de observaciones repetidas simultáneamente de dos magnitudes P y Q, la covarianza asociada a las medias aritméticas $\overline{p}$ y $\overline{q}$ viene dada por

$$s(\overline{p}, \overline{q}) = \frac{1}{n(n-1)} \sum_{j=1}^{n} (p_j - \overline{p})(q_j - \overline{q}) \tag{D.2}$$

y, por substitución, puede calcularse r a partir de la ecuación (D.1).

D3 En el caso de las magnitudes de influencia, cualquier grado de correlación tiene que basarse en la experiencia. Cuando existe correlación, la ecuación (4.1) tiene que substituirse por

$$u^2(y) = \sum_{i=1}^{N} c_i^2 u^2(x_i) + 2\sum_{i=1}^{N-1} \sum_{k=i+1}^{N} c_i c_k u(x_i, x_k) \tag{D.3}$$

donde c_i y c_k son los coeficientes de sensibilidad definidos por la ecuación (4.3) o bien

$$u^2(y) = \sum_{i=1}^{N} u_i^2(y) + 2\sum_{i=1}^{N-1} \sum_{k=i+1}^{N} u_i(y) u_k(y) r(x_i, x_k) \tag{D.4}$$

donde las contribuciones $u_i(y)$ a la incertidumbre típica de la estimación de salida y se derivan de la incertidumbre típica de la estimación de entrada x_i dada por la ecuación (4.2). Debe recordarse que el segundo sumatorio de términos en la ecuación (D.3) o (D.4) puede tener signo negativo.

D4 En la práctica, las magnitudes de entrada están a menudo correlacionadas, ya sea porque en la evaluación de sus valores se utiliza el mismo patrón de referencia físico, el mismo instrumento de medida, los mismos dados de referencia o incluso el mismo método de medida, que tiene una incertidumbre significativa. Sin pérdida de generalidad, supóngase que dos magnitudes de entrada X_1 y X_2 estimadas por x_1 y x_2 dependen del conjunto de variables independientes Q_l (l = 1,2,....L)

$$\begin{aligned} X_1 &= g_1 (Q_1, Q_2, .\,.; Q_L) \\ X_2 &= g_2 (Q_1, Q_2, .\,.; Q_L) \end{aligned} \tag{D.5}$$

aunque es posible que algunas de estas variables no aparezcan en las dos funciones. Las estimaciones x_1 y x_2 de las magnitudes de entrada estarán correlacionadas en cierto grado, incluso aunque las estimaciones q_l (l = 1,2,...,L) no estén correlacionadas. En tal caso, la covarianza $u(x_1,x_2)$ asociada a las estimaciones x_1 y x_2 viene dada por

$$u(x_1, x_2) = \sum_{l=1}^{L} c_{1l} c_{2l} u^2(q_l) \qquad \text{(D.6)}$$

donde c_{1l} y c_{2l} son los coeficientes de sensibilidad derivados de las funciones g_1 y g_2 por analogía con la ecuación (4.3). Puesto que sólo contribuyen a la suma los términos para los que no se anulan los coeficientes de sensibilidad, la covarianza es cero cuando no existe ninguna variable común en las funciones g_1 y g_2. El coeficiente de correlación $r(x_1,x_2)$ asociado a las estimaciones x_1 y x_2 se determina a partir de la ecuación (D.6) conjuntamente con la ecuación (D.1).

D5 El siguiente ejemplo demuestra las correlaciones que existen entre valores atribuidos a dos patrones que se calibran frente al mismo patrón de referencia.

El problema de medida

Los dos patrones X_1 y X_2 se comparan con el patrón de referencia Q_S utilizando un sistema de medida capaz de determinar una diferencia z en sus valores con una incertidumbre típica asociada $u(z)$. Se sabe que el valor q_S del patrón de referencia tiene una incertidumbre típica $u(q_S)$.

Modelo matemático

Las estimaciones x_1 y x_2 dependen del valor q_S del patrón de referencia y de las diferencias observadas z_1 y z_2 conforme a las relaciones

$$\begin{aligned} x_1 &= q_s - z_1 \\ x_2 &= q_s - z_2 \end{aligned} \qquad \text{(D.7)}$$

Incertidumbres típicas y covariancias

Se supone que las estimaciones z_1, z_2 y q_s no están correlacionadas, porque se han determinado en diferentes mediciones. Las incertidumbres típicas se calculan a partir de la ecuación (4.4) y la covarianza asociada a las estimaciones x_1 y x_2 se calcula a partir de la ecuación (D.6), suponiendo que $u(z_1) = u(z_2) = u(z)$,

$$\begin{aligned} u^2(x_1) &= u^2(q_s) + u^2(z) \\ u^2(x_2) &= u^2(q_s) + u^2(z) \\ u(x_1, x_2) &= u^2(q_s) \end{aligned} \qquad \text{(D.8)}$$

El coeficiente de correlación que se deduce de estos resultados es

$$r(x_1, x_2) = \frac{u^2(q_s)}{u^2(q_s) + u^2(z)} \tag{D.9}$$

Su valor está comprendido entre 0 y +1, dependiendo del cociente entre las incertidumbres típicas $u(q_s)$ y $u(z)$.

D6 El caso que se describe en la ecuación (D.5) es un supuesto en el que, con una elección adecuada de la función modelo, puede evitarse la inclusión de la correlación en la evaluación de la incertidumbre típica del mensurando La introducción directa de las variables independientes Q_1 en sustitución de las variables originales X_1 y X_2 en la función modelo f conforme a las ecuaciones de transformación (D.5) genera una nueva función modelo que no contiene ya las variables correlacionadas X_1 y X_2.

D7 No obstante, existen casos en los que no puede evitarse la correlación entre dos magnitudes de entrada X_1 y X_2, por ejemplo cuando se utiliza el mismo instrumento de medida o el mismo patrón de referencia para determinar las estimaciones de entrada x_1 y x_2, pero no se dispone de ecuaciones de transformación a nuevas variables independientes.

Si además no se conoce con exactitud el grado de correlación, puede ser útil valorar la máxima influencia que esta correlación puede tener mediante una estimación del límite superior de la incertidumbre típica del mensurando, que en el caso de que no se tomen en cuenta otras correlaciones adopta la forma

$$u^2(y) \le \left(\left|u_1(y)\right| + \left|u_2(y)\right|\right)^2 + u_r^2(y) \tag{D.10}$$

siendo $u_r(y)$ la contribución a la incertidumbre típica de todas las magnitudes de entrada restantes que se supone que no están correlacionadas.

Nota: La ecuación (D.10) se generaliza fácilmente a los casos de uno o más grupos con dos o más magnitudes de entrada correlacionadas. En este caso ha de introducirse en la ecuación (D.10) una suma del peor de los casos para cada grupo de magnitudes correlacionadas.

ANEXO E Factores de cobertura derivados de los grados efectivos de libertad.

E1 Para determinar el valor de un factor de cobertura *k* correspondiente a una determinada probabilidad de cobertura, es necesario tener en cuenta la fiabilidad de la incertidumbre típica *u(y)* de la estimación de salida *y*. Esto significa que hay que tener en cuenta la fiabilidad con que *u(y)* estima la desviación típica asociada al resultado de la medición. Para una estimación de la desviación típica de una distribución normal, los grados de libertad de esa estimación, que dependen del tamaño de la muestra en la que se basa, son una medida de su fiabilidad. Del mismo modo, una medida satisfactoria de la fiabilidad de la incertidumbre típica asociada a una estimación de salida son sus grados efectivos de libertad ν_{eff}, que se obtienen por aproximación mediante una combinación adecuada de los grados efectivos de libertad de las diferentes contribuciones a la incertidumbre $u_i(y)$.

E2 El procedimiento para calcular un factor de cobertura *k* adecuado cuando se cumplen las condiciones del teorema central del límite incluye las siguientes tres etapas:

(a) Obtener la incertidumbre típica asociada a la estimación de salida según el procedimiento paso a paso descrito en el apartado 7.

(b) Determinar los grados efectivos de libertad ν_{eff} de la incertidumbre típica *u(y)* asociada a la estimación de salida *y* a partir de la fórmula de Welch-Satterthwaite

$$\nu_{\text{eff}} = \frac{u^4(y)}{\sum_{i=1}^{N} \frac{u_i^4(y)}{\nu_i}} , \qquad \text{(E.1)}$$

donde $u_i(y)$ $(i=1,2,...,N)$, definidas en la ecuación (4.2), son las contribuciones a la incertidumbre típica asociada a la estimación de salida y resultante de la incertidumbre típica asociada a la estimación de entrada x_i, que se supone que son mutuamente independientes estadísticamente, y ν_i son los grados efectivos de libertad de la contribución a la incertidumbre típica $u_i(y)$.

Para una incertidumbre típica *u(q)* obtenida mediante una evaluación tipo A tal y como se expone en la subsección 3.1, los grados de libertad vienen dados por $\nu_i = n-1$. Es más problemático asociar grados de libertad a una incertidumbre típica $u(x_i)$ obtenida mediante una evaluación de tipo B. En todo caso, es práctica común realizar estas evaluaciones de manera que se evite toda subestimación. Por ejemplo, si se establecen un límite inferior y otro superior a_- y a_+, generalmente se eligen de forma que se reduzca mucho la probabilidad de que la magnitud en cuestión quede fuera de dichos límites. Siempre que se siga esta práctica, puede considerarse que los grados de libertad de la incertidumbre típica $u(x_i)$ obtenidos por evaluación tipo B pueden tomarse como $\nu_i \to \infty$.

(c) Obtener el factor de cobertura *k* a partir de la tabla de valores E.1 de este Anexo. Esta tabla de valores se basa en una distribución t de Student evaluada para una probabilidad de cobertura del 95,45%. Si ν_{eff} no es un número entero, como ocurre a menudo, se debe truncar ν_{eff} al siguiente número entero más pequeño.

Tabla E.1: Factores de cobertura *k* para diferentes grados efectivos de libertad ν_{eff}.

ν_{eff}	1	2	3	4	5	6	7	8	9	10
k	13,97	4,53	3,31	2,87	2,65	2,52	2,43	2,37	2,32	2,28

ν_{eff}	11	12	13	14	15	16	17	18	19	20
k	2,25	2,23	2,21	2,20	2,18	2,17	2,16	2,15	2,14	2,13

ν_{eff}	25	30	35	40	45	50	∞
k	2,11	2,09	2,07	2,06	2,06	2,05	2,00

SUPPLEMENT 1

Examples

CONTENTS

S1 INTRODUCTION

S1.1 The following examples are chosen to demonstrate the method of evaluating the uncertainty of measurement. More typical and representative examples based on appropriate models have to be developed by special working groups in the different areas. Nevertheless, the examples presented here provide a general guidance on how to proceed.

S1.2 The examples are based on drafts prepared by EAL Expert Groups. These drafts have been simplified and harmonised to make them transparent to laboratory staff in all fields of calibration. It is thus hoped that this set of examples will contribute to a better understanding of the details of setting up the model of evaluation and to the harmonisation of the process of evaluating the uncertainty of measurement, independent of the field of calibration.

S1.3 The contributions and values given in the examples are not intended to imply mandatory or preferred requirements. Laboratories should determine the uncertainty contributions on the basis of the model function they use in the evaluation of the particular calibration they perform and report the evaluated uncertainty of measurement on the calibration certificate they issue. In all the examples given, the conditions stated in section 5 for the use of the standard coverage factor k = 2 are fulfilled.

S1.4 The presentation of the examples follows, in accordance with the step-by-step procedure of section 7 of EAL-R2, a common scheme containing:

- a short descriptive title,
- a general description of the process of measurement,
- the model of evaluation with a list of symbols used,
- an extended listing of input data with short descriptions of how they have been obtained,
- the list of observations and the evaluation of the statistical parameters,
- an uncertainty budget in table form,
- the expanded uncertainty of measurement,
- the reported complete result of measurement.

S1.5 This first supplement to EAL-R2 is intended to be followed by others containing further worked-out examples on the evaluation of uncertainty of measurement in connection with the calibration of instruments. Examples may also be found in EAL Guidance Documents dealing with the calibration of specific types of measurement instruments.

S2 CALBRATION OF A WEIGHT OF NOMINAL VALUE 10 KG

S2.1 The calibration of a weight of nominal value 10 kg of OIML class Ml is carried out by comparison to a reference standard (OIML class F2) of the same nominal value using a mass comparator whose performance characteristics have previously been determined.

S2.2 The unknown conventional mass m_X is obtained from:

$$m_X = m_S + \delta d_D + \delta m + \delta m_C + \delta B \qquad \text{(S2.1)}$$

where:

m_S - conventional mass of the standard,

δm_D - drift of value of the standard since its last calibration,

δm - observed difference in mass between the unknown mass and the standard,

δm_C - correction for eccentricity and magnetic effects,

δB - correction for air buoyancy.

S2.3 **Reference standard (m_S):** The calibration certificate for the reference standard gives a value of 10 000,005 g with an associated expanded uncertainty of 45 mg (coverage factor k = 2).

S2.4 **Drift of the value of the standard (δm_D):** The drift of the value of the reference standard is estimated from previous calibrations to be zero within ±15 mg.

S2.5 **Comparator (δm, δm_C):** A previous evaluation of the repeatability of the mass difference between two weights of the same nominal value gives a pooled estimate of standard deviation of 25 mg. No correction is applied for the comparator, whereas variations due to eccentricity and magnetic effects are estimated to have rectangular limits of ±10 mg.

S2.6 **Air buoyancy (δB):** No correction is made for the effects of air buoyancy, the limits of deviation are estimated to be $\pm 1 \times 10^{-6}$ of the nominal value.

S2.7 **Correlation:** None of the input quantities are considered to be correlated to any significant extent.

S2.8 Measurements: Three observations of the difference in mass between the unknown mass and the standard are obtained using the substitution method and the substitution scheme ABBA ABBA ABBA:

no	**conventional mass**		**reading**		**observed difference**
1	standard		+0,010 g		
	unknown		+0,020 g		
	unknown		+0,025 g		
	standard		+0,015 g		+0,01 g
2	standard		+0,025 g		
	unknown		+0,050 g		
	unknown		+0,055 g		
	standard		+0,020 g		+0,03 g
3	standard		+0,025 g		
	unknown		+0,045 g		
	unknown		+0,040 g		
	standard		+0,020 g		+0,02 g

arithmetic mean: $\overline{\delta m}$ = 0,020 g

pooled estimate of standard deviation: $s_p(\delta m)$ = 25 mg
(obtained from prior evaluation)

standard uncertainty: $u(\delta m) = s(\overline{\delta m}) = \frac{25\ \mathrm{mg}}{\sqrt{3}} = 14{,}4\ \mathrm{mg}$

S2.9 Uncertainty budget (m_X):

quantity X_i	estimate x_i	standard uncertainty $u(x_i)$	probability distribution	sensitivity coefficient c_i	uncertainty contribution $u_i(y)$
m_S	10 000,005 g	22,5 mg	normal	1,0	22,5 mg
δm_D	0,000 g	8,95 mg	rectangular	1,0	8,95 mg
δm	0,020 g	14,4 mg	normal	1,0	14,4 mg
δm_C	0,000 g	5,77 mg	rectangular	1,0	5,77 mg
δB	0,000 g	5,77 mg	rectangular	1,0	5,77 mg
m_X	10 000,025 g				29,3 mg

S2.10 Expanded uncertainty

$U = k \times u(m_X) = 2 \times 29{,}3\ \mathrm{mg} \cong 59\ \mathrm{mg}$

S2.11 Reported result

The measured mass of the nominal 10 kg weight is 10,000 025 kg ±59 mg.
The reported expanded uncertainty of measurement is stated as the standard uncertainty of measurement multiplied by the coverage factor k = 2, which for a normal distribution corresponds to a coverage probability of approximately 95 %.

S3 CALIBRATION OF NOMINAL 10 KΩ STANDARD RESISTOR

S3.1 The resistance of a four-terminal standard resistor is determined by direct substitution using a long-scale digital multimeter (7½ digit DMM) on its resistance range, and a calibrated four-terminal standard resistor of the same nominal value as the item to be calibrated as reference standard. The resistors are immersed in a well stirred oil bath operating at a temperature of 23 °C monitored by a centrally placed mercury-in-glass thermometer. The resistors are allowed to stabilise before the measurement. The four-terminal connectors of each resistor are connected in turn to the terminals of the DMM. It is determined that the measuring current on the 10 kΩ range of the DMM of 100 μA is sufficiently low not to cause any appreciable self-heating of the resistors. The measuring procedure used also ensures that the effects of external leakage resistances on the result of measurement can be considered to be insignificant.

S3.2 The resistance R_X of the unknown resistor is obtained from the relationship:

$$R_X = (R_S + \delta R_D + \delta R_{TS}) r_C \, r - \delta R_{TX} \tag{S3.1}$$

where:

R_S	-	resistance of the reference,
δR_D	-	drift of the resistance of the reference since its last calibration,
δR_{TS}	-	temperature related resistance variation of the reference,
$r = R_{iX}/R_{iS}$	-	ratio of the indicated resistance (index i means 'indicated') for the unknown and reference resistors,
rC	-	*correction factor for parasitic voltages and instrument resolution*
δR_{TX}	-	temperature-related resistance variation of the unknown resistor.

S3.3 **Reference standard (R_S):** The calibration certificate for the reference standard gives a resistance value of 10 000,053 Ω ±5 mΩ (coverage factor k = 2) at the specified reference temperature of 23 °C.

S3.4 **Drift of the value of the standard (δR_D):** The drift of the resistance of the reference resistor since its last calibration is estimated from its calibration history to be +20 mΩ with deviations within ±10 mΩ.

S3.5 **Temperature corrections (δR_{TS}, δR_{TX}):** The temperature of the oil bath is monitored using a calibrated thermometer to be 23,00 °C. Taking into account the metrological characteristics of the thermometer used and of gradients of temperature within the oil bath, the temperature of the resistors is estimated to coincide with the monitored temperature within ±0,055 K. Thus the known value 5×10^{-6} K^{-1} of the temperature coefficient (TC) of the reference resistor gives limits ±2,75 mΩ for the deviation from its resistance value according to calibration, due to a possible deviation from the operating temperature. From the manufacturer's literature, the TC of the unknown resistor is estimated not to exceed 10×10^{-6} K^{-1}, thus the resistance variation of the unknown resistor due to a temperature variation is estimated to be within ±5,5 mΩ.

S3.6 **Resistance measurements (r_C):** Since the same DMM is used to observe both R_{iX} and R_{iS} the uncertainty contributions are correlated but the effect is to reduce the uncertainty and it is only necessary to consider the relative difference in the resistance readings due to systematic effects such as parasitic voltages and instrument resolution (see the mathematical note in paragraph S3.12), which are estimated to have limits of $\pm 0{,}5 \times 10^{-6}$ for each reading. The distribution resulting for the ratio r_C is triangular with expectation 1,000 000 0 and limits $\pm 1{,}0 \times 10^{-6}$.

S3.7 **Correlation:** None of the input quantities are considered to be correlated to any significant extent.

S3.8 **Measurements(r):** Five observations are made to record the ratio r:

No.	observed ratio
1	1,000 010 4
2	1,000 010 7
3	1,000 010 6
4	1,000 010 3
5	1,000 010 5

arithmetic mean: $\bar{r} = 1{,}000\,010\,5$

experimental standard deviation: $s(r) = 0{,}158 \times 10^{-6}$

standard uncertainty: $u(r) = s(\bar{r}) = \dfrac{0{,}158 \times 10^{-6}}{\sqrt{5}} = 0{,}0707 \times 10^{-6}$

S3.9 **Uncertainty budget (R_X):**

quantity X_i	estimate x_i	standard uncertainty $u(x_i)$	probability distribution	sensitivity coefficient c_i	uncertainty contribution $u_i(y)$
R_S	10 000,053 Ω	2,5 mΩ	normal	1,0	2,5 mΩ
δR_D	0,020 Ω	5,8 mΩ	rectangular	1,0	5,8 mΩ
δR_{TS}	0,000 Ω	1,6 mΩ	rectangular	1,0	1,6 mΩ
δR_{TX}	0,000 Ω	3,2 mΩ	rectangular	1,0	3,2 mΩ
r_C	1,000 000 0	$0{,}41 \times 10^{-6}$	triangular	10 000 Ω	4,1 mΩ
r	1,000 010 5	$0{,}07 \times 10^{-6}$	normal	10 000 Ω	0,7 mΩ
R_X	10 000,178 Ω				8,33 mΩ

S3.10 Expanded uncertainty:

$$U = k \times u(R_X) = 2 \times 8{,}33\ \text{m}\Omega \cong 17\ \text{m}\Omega$$

S3.11 Reported result: The measured value of the nominal 10 kΩ resistor, at a measuring temperature of 23,00 °C and a measuring current of 100 μA, is (10 000,178 ±0,017) Ω.

The reported expanded uncertainty of measurement is stated as the standard uncertainty of measurement multiplied by the coverage factor k = 2, which for a normal distribution corresponds to a coverage probability of approximately 95 %.

S3.12 Mathematical note on the standard uncertainty of measurement of the ratio of indicated resistance values: The unknown and the reference resistors have nearly the same resistance. Within the usual linear approximation in the deviations, the values causing the DMM indications R_{iX} and R_{iS} are given by

$$\begin{aligned} R_X' &= R_{iX}\left(1 + \frac{\delta R_X'}{R}\right) \\ R_S' &= R_{iS}\left(1 + \frac{\delta R_S'}{R}\right) \end{aligned} \qquad \text{(S3.2)}$$

with R being the nominal value of the resistors and $\delta R_X'$ and $\delta R_S'$ the unknown deviations. The resistance ratio deduced from these expressions is

$$\frac{R_X'}{R_S'} = r r_C \qquad \text{(S3.3)}$$

with the ratio of the indicated resistance for the unknown and the reference resistor

$$r = \frac{R_{iX}}{R_{iS}} \qquad \text{(S3.4)}$$

and the correction factor (linear approximation in the deviations)

$$r_C = 1 + \frac{\delta R_X' - \delta R_S'}{R} \qquad \text{(S3.5)}$$

Because of the fact that the difference of the deviations enters into equation (S3.5), correlated contributions of systematic effects resulting from the internal scale of the DMM do not influence the result. The standard uncertainty of the correction factor is determined only by uncorrelated deviations resulting from the parasitic effects and the resolution of the DMM. Assuming that $u(\delta R_X') = u(\delta R_S') = u(\delta R')$, it is given by the expression

$$u^2(r_C) = 2\frac{u^2(\delta R')}{R^2} \qquad \text{(S3.6)}$$

S4 CALIBRATION OF A GAUGE BLOCK OF NOMINAL LENGTH 50 MM

S4.1 The calibration of the grade 0 gauge block (ISO 3650) of 50 mm nominal length is carried out by comparison using a comparator and a calibrated gauge block of the same nominal length and the same material as reference standard. The difference in central length is determined in vertical position of the two gauge blocks using two length indicators contacting the upper and lower measuring faces. The actual length l_X' of the gauge block to be calibrated is related to the actual length l_S' of the reference standard by the equation

$$l_X' = l_S' + \delta l \qquad \text{(S4.1)}$$

with δl being the measured length difference. l_X' and l_S' are the lengths of the gauge blocks under measurement conditions, in particular at a temperature which, on account of the uncertainty in the measurement of laboratory temperature, may not be identical with the reference temperature for length measurements.

S4.2 The length l_X of the unknown gauge block at the reference temperature is obtained from the relationship:

$$l_X = l_S + \delta l_D + \delta l + \delta l_C - L(\bar{\alpha} \times \delta t + \delta \alpha \times \Delta \bar{t}) - \delta l_V \qquad \text{(S4.2)}$$

where:

l_S	-	length of the reference gauge block at the reference temperature t_0 = 20 °C according to its calibration certificate;
δl_D	-	change of the length of the reference gauge block since its last calibration due to drift;
δl	-	observed difference in length between the unknown and the reference gauge block;
δl_C	-	correction for non-linearity and offset of the comparator;
L	-	nominal length of the gauge blocks considered;
$\bar{\alpha} = (\alpha_X + \alpha_S)/2$	-	average of the thermal expansion coefficients of the unknown and reference gauge blocks;
$\delta t = (t_X - t_S)$	-	temperature difference between the unknown and reference gauge blocks;
$\delta\alpha = (\alpha_X - \alpha_S)$	-	difference in the thermal expansion coefficients between the unknown and the reference gauge blocks;
$\Delta\bar{t} = (t_X + t_S)/2 - t_0$	-	deviation of the average temperature of the unknown and the reference gauge blocks from the reference temperature;
δl_V	-	correction for non-central contacting of the measuring faces of the unknown gauge block.

S4.3 **Reference standard (l_S):** The length of the reference gauge block together with the associated expanded uncertainty of measurement is given in the calibration certificate of a set of gauge blocks as 50,000 02 mm ±30 nm (coverage factor k = 2).

S4.4 **Drift of the standard (δl_D):** The temporal drift of the length of the reference gauge block is estimated from previous calibrations to be zero with limits ±30 nm. General experience with gauge blocks of this type suggests that zero drift is most probable and that a triangular probability distribution can be assumed.

S4.5 **Comparator (δl_C):** The comparator has been verified to meet the specifications stated in EAL-G21. From this, it can be ascertained that for length differences D up to ±10 μm corrections to the indicated length difference are within the limits $\pm$(30 nm +0,02·$|D|$). Taking into account the tolerances of the grade 0 gauge block to be calibrated and the grade K reference gauge block, the maximum length difference will be within ±1 μm leading to limits of ±32 nm for non-linearity and offset corrections of the comparator used.

S4.6 **Temperature corrections ($\bar{\alpha}$, δt, $\delta\alpha$, $\Delta\bar{t}$):** Before calibration, care is taken to ensure that the gauge blocks assume ambient temperature of the measuring room. The remaining difference in temperature between the standard and the gauge block to be calibrated is estimated to be within ±0,05 K. Based on the calibration certificate of the reference gauge block and the manufacturer's data for the gauge block to be calibrated the linear thermal expansion coefficient of the steel gauge blocks is assumed to be within the interval $(11{,}5 \pm 1{,}0)\times 10^{-6}$ °C^{-1}. Combining the two rectangular distributions the difference in linear thermal expansion coefficient is triangularly distributed within the limits $\pm 2\times 10^{-6}$ °C^{-1}. The deviation of the mean temperature of measurement from the reference temperature t_0 = 20 °C is estimated to be within ±0,5 °C. The best estimates of the difference in linear expansion coefficients and the deviations of the mean temperature from the reference temperature are zero. Therefore second order terms have to be taken into account in the evaluation of their uncertainty contribution resulting in the product of standard uncertainties associated with the factors of the product term $\delta\alpha \times \Delta\bar{t}$ in equation (S4.2) (see the mathematical note in paragraph S4.13, eq. (S4.5)). The final standard uncertainty is $u(\delta\alpha \times \Delta\bar{t}) = 0{,}236 \times 10^{-6}$.

S4.7 **Variation in length (δl_V):** For gauge blocks of grade 0 the variation in length determined from measurements at the centre and the four corners has to be within ±0,12 μm (ISO 3650). Assuming that this variation occurs on the measuring faces along the short edge of length 9 mm and that the central length is measured inside a circle of radius 0,5 mm, the correction due to central misalignment of the contacting point is estimated to be within ±6,7 nm.

S4.8 **Correlation:** None of the input quantities are considered to be correlated to any significant extent.

S4.9 Measurements (δl): The following observations are made for the difference between the unknown gauge block and the reference standard, the comparator being reset using the reference standard before each reading.

obs. no.	obs. value
1	-100 nm
2	-90 nm
3	-80 nm
4	-90 nm
5	-100 nm

arithmetic mean: $\overline{\delta l} = -94 \text{ nm}$

pooled estimate of standard deviation: $s_p(\delta l) = 12 \text{ nm}$
(obtained from prior evaluation)

standard uncertainty: $u(\delta l) = s(\overline{\delta l}) = \dfrac{12 \text{ nm}}{\sqrt{5}} = 5{,}37 \text{ nm}$

The pooled estimate of the standard deviation has been taken from the tests made to confirm compliance of the comparator used with the requirements of EAL-G21.

S4.10 Uncertainty budget (δl_X):

quantity X_i	estimate x_i	standard uncertainty $u(x_i)$	probability distribution	sensitivity coefficient c_i	uncertainty contribution $u_i(y)$
l_S	50,000 020 mm	15 nm	normal	1,0	15,0 nm
δl_D	0 mm	17,3 nm	triangular	1,0	17,3 nm
δl	-0,000 094 mm	5,37 nm	normal	1,0	5,37 nm
δl_C	0 mm	18,5 nm	rectangular	1,0	18,5 nm
δt	0 °C	0,0289 °C	rectangular	-575 nm°C^{-1}	-16,6 nm
$\delta\alpha \times \Delta\bar{t}$	0	$0{,}236\times10^{-6}$	special	50 mm	-11,8 nm
δl_V	0 mm	3,87 nm	rectangular	-1,0	-3,87 nm
l_X	49,999 926 mm				36,4 nm

S4.11 Expanded uncertainty

$$U = k \times u(l_X) = 2 \times 36{,}4 \text{ nm} \cong 73 \text{ nm}$$

S4.12 Reported result

The measured value of the nominal 50 mm gauge block is 49,999 926 mm ±73 nm.

The reported expanded uncertainty of measurement is stated as the standard uncertainty of measurement multiplied by the coverage factor k = 2, which for a normal distribution corresponds to a coverage probability of approximately 95 %.

S4.13 Mathematical note on the standard uncertainty of measurement of the product of two quantities with zero expectation: If a product of two quantities is considered, the usual method of evaluation of uncertainty contributions based on the linearisation of the model function has to be modified if one or both of the expectations of the factors in the product are zero. If the factors in the product are statistically independent with non-zero expectations, the square of the relative standard uncertainty of measurement (relative variance) associated with the product can be expressed without any linearisation by the squares of the relative standard uncertainties associated with the estimates of the factors:

$$w^2(x_1 \times x_2) = w^2(x_1) + w^2(x_2) + w^2(x_1) \times w^2(x_2) \tag{S4.2}$$

Using the definition of the relative standard uncertainty of measurement this expression is easily transformed into the general relation

$$u^2(x_1 \times x_2) = x_2^2\, u^2(x_1) + x_1^2\, u^2(x_2) + u^2(x_1) \times u^2(x_2) \tag{S4.3}$$

If the standard uncertainties $u(x_1)$ and $u(x_2)$ associated with the expectations x_1 and x_2 are much smaller than the moduli of the respective expectation values the third term on the right side may be neglected. The resulting equation represents the case described by the usual method based on the linearisation of the model function.

If, however, one of the moduli of the expectation values, for example $|x_2|$, is much smaller than the standard uncertainty $u(x_2)$ associated with this expectation or even zero, the product term involving this expectation may be neglected on the right side of equation (S4.3), but not the third term. The resulting equation is

$$u^2(x_1 \times x_2) \cong x_1^2\, u^2(x_2) + u^2(x_1) \times u^2(x_2) \tag{S4.4}$$

If both moduli of the expectation values are much smaller than their associated standard uncertainties or even zero, only the third term in equation (S4.3) gives a significant contribution:

$$u^2(x_1 \times x_2) \cong u^2(x_1) \times u^2(x_2) \tag{S4.5}$$

S5 CALIBRATION OF A TYPE N THERMOCOUPLE AT 1000°C

S5.1 A type N thermocouple is calibrated by comparison with two reference thermocouples of type R in a horizontal furnace at a temperature of 1000 °C. The emfs generated by the thermocouples are measured using a digital voltmeter through a selector/reversing switch. All thermocouples have their reference junctions at 0 °C. The thermocouple to be calibrated is connected to the reference point using compensating cables. Temperature values are give in the t_{90} temperature scale.

S5.2 The temperature t_X of the hot junction of the thermocouple to be calibrated is

$$\begin{aligned} t_\mathrm{X} &= t_\mathrm{S}\Big(V_\mathrm{iS} + \delta V_\mathrm{iS1} + \delta V_\mathrm{iS2} + \delta V_\mathrm{R} - \frac{\delta t_\mathrm{0S}}{C_\mathrm{S0}}\Big) + \delta t_\mathrm{D} + \delta t_\mathrm{F} \\ &\cong t_\mathrm{S}(V_\mathrm{iS}) + C_\mathrm{S} \times \delta V_\mathrm{iS1} + C_\mathrm{S} \times \delta V_\mathrm{iS2} + C_\mathrm{S} \times \delta V_\mathrm{R} - \frac{C_\mathrm{S}}{C_\mathrm{S0}} \delta t_\mathrm{0S} + \delta t_\mathrm{D} + \delta t_\mathrm{F} \end{aligned} \tag{S5.1}$$

S5.3 The voltage V_X across the thermocouple wires with the cold junction at 0 °C during calibration is

$$\begin{aligned} V_X(t) &\cong V_X(t_X) + \frac{\Delta t}{C_X} - \frac{\delta t_{0X}}{C_{X0}} \\ &= V_{iX} + \delta V_{iX1} + \delta V_{iX2} + \delta V_R + \delta V_{LX} + \frac{\Delta t}{C_X} - \frac{\delta t_{0X}}{C_{X0}} \end{aligned} \tag{S5.2}$$

where:

$t_S(V)$	-	temperature of the reference thermometer in terms of voltage with cold junction at 0 °C. The function is given in the calibration certificate;
V_{iS}, V_{iX}	-	indication of the voltmeter;
δV_{iS1}, δV_{iX1}	-	voltage corrections obtained from the calibration of the voltmeter;
δV_{iS2}, δV_{iX2}	-	voltage corrections due to the limited resolution of the voltmeter;
δV_R	-	voltage correction due to contact effects of the reversing switch;
δt_{0S}, δt_{0X}	-	temperature corrections due to the deviation of the reference temperatures from 0 °C;
C_S, C_X	-	sensitivities of the thermocouples for voltage at the measuring temperature of 1000 °C;
C_{S0}, C_{X0}	-	sensitivities of the thermocouples for voltage at the reference temperature of 0 °C;
δt_D	-	change of the values of the reference thermometers since their last calibration due to drift;
δt_F	-	temperature correction due to non-uniformity of the temperature of the furnace;
t	-	temperature at which the thermocouple is to be calibrated (calibration point);
$\Delta t = t - t_X$	-	deviation of the temperature of the calibration point from the temperature of the furnace;
δV_{LX}	-	voltage correction due to the compensating cables.

S5.4 The reported result is the output emf of the thermocouple at the temperature of its hot junction. Because the measurement process consists of two steps — determination of the temperature of the furnace and determination of emf of the thermocouple to be calibrated — the evaluation of the uncertainty of measurement is split in two parts.

S5.5 **Reference standards ($t_S(V)$):** The reference thermocouples are supplied with calibration certificates that relate the temperature at their hot junction with their cold junction at 0 °C to the voltage across their wires. The expanded uncertainty of measurement at 1000 °C is U = 0,3 °C (coverage factor k = 2).

S5.6 **Calibration of the voltmeter (δV_{iS1}, δV_{iX1}):** The voltmeter has been calibrated. Corrections to the measured voltages are made to all results. The calibration certificate gives a constant expanded uncertainty of measurement for voltages smaller than 50 mV of U = 2,0 µV (coverage factor k = 2).

S5.7 **Resolution of the voltmeter (δV_{iS2}, δV_{iX2}):** A 4½ digit microvoltmeter has been used in its 10 mV range resulting in resolution limits of ±0,5 µV at each indication.

S5.8 **Parasitic voltages (δV_R):** Residual parasitic offset voltages due to the switch contacts have been estimated to be zero within ±2 µV.

S5.9 **Reference temperatures (δt_{0S}, δt_{0X}):** The temperature of the reference point of each thermocouple is known to be 0 °C within ±0,1 °C.

S5.10 **Voltage sensitivities (C_S, C_X, C_{S0}, C_{X0}):** The voltage sensitivities of the thermocouples have been taken from reference tables:

	1000 °C	0 °C
reference thermocouple	C_S = 0,077 °C/µV	C_{S0} = 0,189 °C/µV
unknown thermocouple	C_X = 0,026 °C/µV	C_{S0} = 0,039 °C/µV

S5.11 **Drift of the reference standard (δt_D):** From previous calibrations the drift of the reference standards are estimated to be zero within the limits ±0,3 °C.

S5.12 **Temperature gradients (δt_F):** The temperature gradients inside the furnace have been measured. At 1000 °C, deviations from non-uniformity of temperature in the region of measurement are within ±1 °C.

S5.13 **Compensating cables (δV_{LX}):** The compensating cables have been investigated in the range 0 °C to 40 °C. From this, the voltage differences between the cables and the thermocouple wires are estimated to be within ±5 µV.

S5.14 **Measurements (V_{iS}, $t_S(V_{iS})$, V_{iX}):** The indications of the voltmeter are recorded in the following operational procedure which gives four readings for every thermocouple and reduces the effects of temperature drift in the thermal source and of parasitic thermal voltages in the measuring circuit:

1st cycle:

1st standard, unknown thermocouple, 2nd standard,

2nd standard, unknown thermocouple, 1st standard.

Reversion of polarity.

2nd cycle:

1st standard, unknown thermocouple, 2nd standard,

2nd standard, unknown thermocouple, 1st standard.

S5.15 The procedure requires that the difference between the two reference standards must not exceed ±0,3 °C. If the difference is not within these limits the observations have to be repeated and/or the reasons for such a large difference have to be investigated.

Thermocouple	**1st reference**	**Unknown**	**2nd reference**
Indicated voltage, corrected	+10500 µV	+36245 µV	+10503 µV
	+10503 µV	+36248 µV	+10503 µV
	-10503 µV	-36248 µV	-10505 µV
	-10504 µV	-36251 µV	-10505 µV
Mean voltage	10502,5 µV	36248 µV	10504 µV
Temperature of the hot junction	1000,4 °C		1000,6 °C
Temperature of the furnace		1000,5 °C	

S5.16 From the four readings on each thermocouple given in the table above, the mean value of the voltages of each thermocouple is deduced. The voltage values of the reference thermocouples are converted into temperature values by means of the temperature-voltage relations stated in their calibration certificates. The observed temperature values are highly correlated (correlation factor nearly one). Therefore, by taking their mean value, they are combined to one observation only, which is the temperature of the furnace at the location of the thermocouple to be calibrated. In a similar way, one observation of the voltage of the thermocouple to be calibrated has been extracted. In order to evaluate the uncertainty of measurement associated with these observations, a series of ten measurements has been previously undertaken at the same temperature of operation. It gave a pooled estimate of standard deviation for the temperature of the furnace and the voltage of the thermocouple to be calibrated.

The respective standard uncertainties of measurement of the observed quantities are:

pooled estimate of standard deviation: $s_p(t_S) = 0{,}10\ °C$

standard uncertainty: $u(t_S) = \dfrac{s_p(t_S)}{\sqrt{1}} = 0{,}10\ °C$

pooled estimate of standard deviation: $s_p(V_{iX}) = 1{,}6\ \mu V$

standard uncertainty: $u(V_{iX}) = \dfrac{s_p(V_{iX})}{\sqrt{1}} = 1{,}6\ \mu V$

S5.17 Uncertainty budget (temperature t_X of the furnace):

quantity X_i	estimate x_i	standard uncertainty $u(x_i)$	probability distribution	sensitivity coefficient c_i	uncertainty contribution $u_i(y)$
t_S	1000,5 °C	0,10 °C	normal	1,0	0,10 °C
δV_{iS1}	0 µV	1,00 µV	normal	0,077 °C/µV	0,077 °C
δV_{iS2}	0 µV	0,29 µV	rectangular	0,077 °C/µV	0,022 °C
δV_R	0 µV	1,15 µV	rectangular	0,077 °C/µV	0,089 °C
δt_{0S}	0 °C	0,058 °C	rectangular	-0,407	-0,024 °C
δt_S	0 °C	0,15 °C	normal	1,0	0,15 °C
δt_D	0 °C	0,173 °C	rectangular	1,0	0,173 °C
δt_F	0 °C	0,577 °C	rectangular	1,0	0,577 °C
t_X	1000,5 °C				0,641 °C

S5.18 Uncertainty budget (emf V_X of the thermocouple to be calibrated):

The standard uncertainty of measurement associated with the temperature deviation of the calibration point from the temperature of the furnace is the standard uncertainty of measurement associated with the temperature of the furnace because the temperature point is a defined value (exactly known).

quantity X_i	estimate x_i	standard uncertainty $u(x_i)$	probability distribution	sensitivity coefficient c_i	uncertainty contribution $u_i(y)$
V_{iX}	36 248 µV	1,60 µV	normal	1,0	1,60 µV
δV_{iX1}	0 µV	1,00 µV	normal	1,0	1,00 µV
δV_{iX2}	0 µV	0,29 µV	rectangular	1,0	0,29 µV
δV_R	0 µV	1,15 µV	rectangular	1,0	1,15 µV
δV_{LX}	0 µV	2,9 µV	rectangular	1,0	2,9 µV
Δt	0,5 °C	0,641 °C	normal	38,5 µV/°C	24,5 µV
δt_{0X}	0 °C	0,058 °C	rectangular	-25,6 µV/°C	-1,48 µV
V_X	36 229 µV				25,0 µV

S5.19 Expanded uncertainties

The expanded uncertainty associated with the measurement of the temperature of the furnace is

$$U = k \times u(t_X) = 2 \times 0{,}641\ °C \cong 1{,}3\ °C$$

The expanded uncertainty associated with the emf value of the thermocouple to be calibrated is

$$U = k \times u(V_X) = 2 \times 25{,}0\ \mu V \cong 50\ \mu V$$

S5.20 Reported result

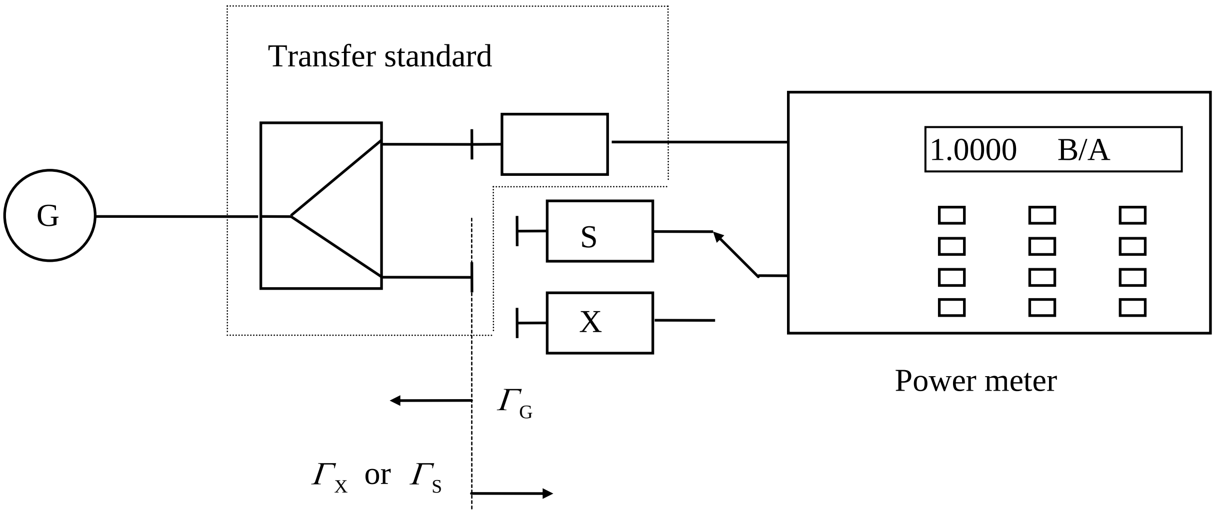
The type N thermocouple shows, at the temperature of 1000,0 °C with its cold junction at a temperature of 0 °C, an emf of 36 230 µV ±50 µV.

The reported expanded uncertainty of measurement is stated as the standard uncertainty of measurement multiplied by the coverage factor $k = 2$, which for a normal distribution corresponds to a coverage probability of approximately 95 %.

S6 CALIBRATION OF A POWER SENSOR AT A FREQUENCY OF 19 GHZ

S6.1 The measurement involves the calibration of an unknown power sensor with respect to a calibrated power sensor used as a reference by substitution on a stable transfer standard of known small reflection coefficient. The measurement is made in terms of calibration factor, which is defined as the ratio of incident power at the reference frequency of 50 MHz to the incident power at the calibration frequency under the condition that both incident powers give equal power sensor response. At each frequency, one determines the (indicated) ratio of the power for the sensor to be calibrated, respectively the reference sensor and the internal sensor that forms part of the transfer standard, using a dual power meter with ratio facility.

S6.2 Schematic of the measuring system



S6.3 The quantity K, termed 'calibration factor' by some manufacturers, is defined as:

$$K = \frac{P_{\mathrm{Ir}}}{P_{\mathrm{Ic}}} = \frac{(1+|\Gamma_{\mathrm{r}}|^2)P_{\mathrm{Ar}}}{(1+|\Gamma_{\mathrm{c}}|^2)P_{\mathrm{Ac}}} \tag{S6.1}$$

for the equal power meter indication

where:

P_{Ir}	-	incident power at the reference frequency (50 MHz),
P_{Ic}	-	incident power at the calibration frequency,
Γ_{r}	-	voltage reflection coefficient of the sensor at the reference frequency
Γ_{c}	-	voltage reflection coefficient of the sensor at the calibration frequency
P_{Ar}	-	power absorbed by the sensor at the reference frequency
P_{Ac}	-	power absorbed by the sensor at the calibration frequency

S6.4 The calibration factor of the unknown sensor is obtained from the relationship

$$K_{\mathrm{X}} = (K_{\mathrm{S}} + \delta K_{\mathrm{D}}) \frac{M_{\mathrm{Sr}} M_{\mathrm{Xc}}}{M_{\mathrm{Sc}} M_{\mathrm{X}r}} p_{\mathrm{Cr}} p_{\mathrm{Cc}} p \tag{S6.2}$$

where:

K_{S}	-	calibration factor of the reference power sensor;
δK_{D}	-	change of the calibration factor of the reference power sensor since its last calibration due to drift;
M_{Sr}	-	mismatch factor of reference sensor at the reference frequency;
M_{Sc}	-	mismatch factor of standard sensor at the calibration frequency;
M_{Xr}	-	mismatch factor of sensor to be calibrated at the reference frequency;
M_{Xc}	-	mismatch factor of sensor to be calibrated at the calibration frequency;
p_{Cr}	-	correction of the observed ratio for non-linearity and limited resolution of the power meter at power ratio level of the reference frequency;
p_{Cc}	-	correction of the observed ratio for non-linearity and limited resolution of the power meter at power ratio level of the calibration frequency;
$p = \frac{p_{\mathrm{Sr}} p_{\mathrm{Xc}}}{p_{\mathrm{Sc}} p_{\mathrm{Xr}}}$	-	observed ratio of power ratios derived from:
p_{Sr}	-	indicated power ratio for the reference sensor at the reference frequency;
p_{Sc}	-	indicated power ratio for the reference sensor at the calibration frequency;
p_{Xr}	-	indicated power ratio for the sensor to be calibrated at the reference frequency;
p_{Xc}	-	indicated power ratio for the sensor to be calibrated at the calibration frequency.

S6.5 **Reference sensor (K_{S}):** The reference sensor was calibrated six months before the calibration of the unknown power sensor. The value of the calibration factor, given in the calibration certificate, is (95,7±1,1) % (coverage factor k = 2), which may also be expressed as 0,957±0,011.

S6.6 **Drift of the standard (δK_D):** The drift of the calibration factor of the reference standard is estimated from annual calibrations to be -0,002 per year with deviations within ±0,004. From these values, the drift of the reference sensor, which was calibrated half a year ago, is estimated to equal -0,001 with deviations within ±0,002.

S6.7 **Linearity and resolution of the power meter (p_{Cr}, p_{Cc}):** The expanded uncertainty of 0,002 (coverage factor k = 2) is assigned to the power meter readings at the power ratio level of the reference frequency and of 0,0002 (coverage factor k = 2) at the power ratio level of calibration frequency due to non-linearity of the power meter used. These values have been obtained from previous measurements. Since the same power meter has been used to observe both p_S and p_X, the uncertainty contributions at the reference as well at the calibration frequency are correlated. Because power ratios at both frequencies are considered, the effect of the correlations is to reduce the uncertainty. Thus, only the relative difference in the readings due to systematic effects should be taken into account (see the mathematical note in paragraph S3.12), resulting in a standard uncertainty of 0,00142 associated with the correction factor p_{Cr} and 0,000142 with the correction factor p_{Cc}.

The expanded uncertainty of measurement stated for the readings of the power meter contains linearity and resolution effects. The linearity effects are correlated whereas the resolution effects are uncorrelated. As shown in S3.12, building the power ratio cancels the influence of correlations and gives a reduced standard uncertainty of measurement to be associated with the ratio. In the calculations above, however, the separated correlated and uncorrelated contributions are not known and the values given are upper bounds for the standard uncertainty of measurement associated with ratios. The uncertainty budget finally shows that the contributions arising from these ratios are insignificant, i.e. the approximations are justified.

S6.8 **Mismatch factors (M_{Sr}, M_{Sc}, M_{Xr} M_{Xc}):** As the transfer standard system is not perfectly matched and the phase of the reflection coefficients of the transfer standard, the unknown and the standard power sensors are not known, there will be an uncertainty due to mismatch for each sensor at the reference frequency and at the calibration frequency. The corresponding limits of deviation have to be calculated for the reference and the calibration frequencies from the relationship:

$$M_{S,X} = 1 \pm 2|\Gamma_G||\Gamma_{S,X}| \tag{S6.3}$$

where the magnitudes of the reflection coefficients of the transfer standard, the reference sensor and the sensor to be calibrated are:

	50 MHz	18 GHz
$\|\Gamma_G\|$	0,02	0,07
$\|\Gamma_S\|$	0,02	0,10
$\|\Gamma_X\|$	0,02	0,12

The probability distribution of the individual contributions is U-shaped. This is taken into account by replacing the factor 1/3 for a rectangular distribution by 1/2 in calculating the variance from the square of the half-width determined from the limits. The standard uncertainty due to mismatch is therefore obtained from:

$$u(M_{\mathrm{S,X}}) = \frac{2|\Gamma_{\mathrm{G}}||\Gamma_{\mathrm{S}}|}{\sqrt{2}} \qquad \text{(S6.4)}$$

Note: The values of the reflection coefficients are the results of measurements which are themselves subject to uncertainty. This is accounted for by adding the square root of the sum of the uncertainty of measurement squared and the measured value squared.

S6.9 **Correlation:** None of the input quantities are considered to be correlated to any significant extent.

S6.10 **Measurements (*p*):** Three separate readings are made which involve disconnection and reconnection of both the reference sensor and the sensor to be calibrated on the transfer standard to take connector repeatability into account. The power meter readings used to calculate the observed power ratio *p* are as follows:

obs. no	p_{Sr}	p_{Sc}	p_{Xr}	p_{Xc}	p
1	1,0001	0,9924	1,0001	0,9698	0,9772
2	1,0000	0,9942	1,0000	0,9615	0,9671
3	0,9999	0,9953	1,0001	0,9792	0,9836

arithmetic mean: $\overline{p} = 0{,}9760$

experimental standard deviation: $s(p) = 0{,}0083$

standard uncertainty: $u(p) = s(\overline{p}) = \dfrac{0{,}0083}{\sqrt{3}} = 0{,}0048$

S6.11 **Uncertainty budget (*K*$_{\mathrm{X}}$):**

quantity X_i	estimate x_i	standard uncertainty $u(x_i)$	probability distribution	sensitivity coefficient c_i	uncertainty contribution $u_i(y)$
K_{S}	0,957	0,0055	normal	0,976	0,00537
δK_{D}	-0,001	0,0012	rectangular	0,976	0,00113
M_{Sr}	1,000	0,0006	U-shaped	0,933	0,00053
M_{Sc}	1,000	0,0099	U-shaped	-0,933	0,00924
M_{Xr}	1,000	0,0006	U-shaped	-0,933	-0,00053
M_{Xc}	1,000	0,0119	U-shaped	0,933	0,01110
p_{Cr}	1,000	0,0014	normal	0,933	0,00132
p_{Cc}	1,000	0,0001	normal	0,933	0,00013
p	0,976	0,0048	normal	0,956	0,00459
K_{X}	0,933				0,01623

S6.12 **Expanded uncertainty**:

$U = k \times u(K_{\mathrm{X}}) = 2 \times 0{,}01623 \cong 0{,}032$

S6.13 Reported result:

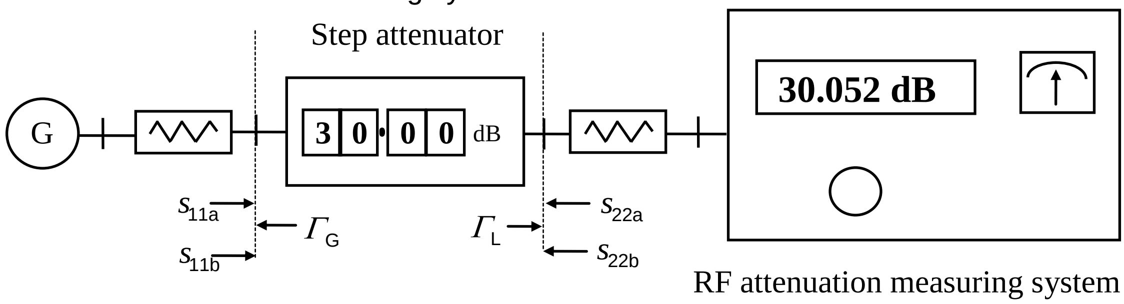
The calibration factor of the power sensor at 18 GHz is 0,933 ±0,032, which may also be expressed as (93,3±3,2) %.

The reported expanded uncertainty of measurement is stated as the standard uncertainty of measurement multiplied by the coverage factor k = 2, which for a normal distribution corresponds to a coverage probability of approximately 95 %.

S7 CALIBRATION OF A COAXIAL STEP ATTENUATOR AT A SETTING OF 30 DB (INCREMENTAL LOSS)

S7.1 The measurement involves the calibration of a coaxial step attenuator at 10 GHz using an attenuation measuring system containing a calibrated step attenuator which acts as the attenuation reference. The method of measurement involves the determination of the attenuation between matched source and matched load. In this case the unknown attenuator can be switched between settings of 0 dB and 30 dB and it is this change (called incremental loss) that is determined in the calibration process. The attenuation measuring system has a digital readout and an analogue null detector which is used to indicate the balance condition.

S7.2 Schematic of the measuring system



S7.3 The attenuation L_X of the attenuator to be calibrated is obtained from the relation:

$$L_X = L_S + \delta L_S + \delta L_D + \delta L_M + \delta L_K + \delta L_{ib} - \delta L_{ia} + \delta L_{0b} - \delta L_{0a} \qquad \text{(S7.1)}$$

where:

$L_S = L_{ib} - L_{ia}$	-	attenuation difference of reference attenuator derived from:
L_{ia}	-	indicated attenuation with the attenuator to be calibrated, set at 0 dB;
L_{ib}	-	indicated attenuation with the attenuator to be calibrated, set at 30 dB;
δL_S	-	correction obtained from the calibration of the reference attenuator;
δL_D	-	change of the attenuation of the reference attenuator since its last calibration due to drift;
δL_M	-	correction due to mismatch loss;
δL_K	-	correction for leakage signals between input and output of the attenuator to be calibrated due to imperfect isolation;
δL_{ia}, δL_{ib}	-	corrections due to the limited resolution of the reference detector at 0 dB and 30 dB settings;
δL_{0a}, δL_{0b}	-	corrections due to the limited resolution of the null detector at 0 dB and 30 dB settings.

S7.4 **Reference attenuator (δL_S):** The calibration certificate for the reference attenuator gives a value of attenuation for the 30,000 dB setting at 10 GHz of 30,003 dB with an associated expanded uncertainty of 0,005 dB (coverage factor $k = 2$). The correction of +0,003 dB with the associated expanded uncertainty of 0,005 dB (coverage factor $k = 2$) is considered to be valid for attenuation settings of the reference attenuator that differ not more than ±0,1 dB from the calibrated setting of 30,000 dB.

S7.5 **Drift of the reference (δL_D):** The drift of the attenuation of the reference attenuator is estimated from its calibration history to be zero with limits ±0,002 dB.

S7.6 **Mismatch loss (δL_M):** The reflection coefficients of the source and the load at the insertion point of the attenuator to be calibrated have been optimised by impedance matching to as low magnitudes as possible. Their magnitudes and the magnitudes of the scattering coefficients of the attenuator to be calibrated have been measured but their phase remains unknown. Without any phase information, a correction for mismatch error cannot be made, but the standard uncertainty (in dB) due to the incomplete knowledge of the match is estimated from the relationship [1]:

$$u(\delta L_M) = \frac{8{,}686}{\sqrt{2}} \sqrt{|\Gamma_S|^2 (|s_{11a}|^2 + |s_{11b}|^2) + |\Gamma_L|^2 (|s_{22a}|^2 + |s_{22b}|^2) + |\Gamma_S|^2 \times |\Gamma_L|^2 (|s_{21a}|^4 + |s_{21b}|^4)} \tag{S7.2}$$

with the source and load reflection coefficients

$\Gamma_L = 0{,}03$ and $\Gamma_S = 0{,}03$

and the scattering coefficients of the attenuator to be calibrated at 10 GHz

	0 dB	30 dB
s_{11}	0,05	0,09
s_{22}	0,01	0,01
s_{21}	0,95	0,031

as $u(\delta L_M) = 0{,}02$ dB.

Note: The values of scattering and reflection coefficients are the results of measurements which are themselves not exactly known. This is accounted for by adding the square root of the sum of uncertainty of measurement squared and the measured value squared.

S7.7 **Leakage correction (δL_K):** Leakage signals through the attenuator to be calibrated have been estimated from the measurements at 0 dB setting to be at least 100 dB below the measurement signal. The correction for leakage signals is estimated from these findings to be within ±0,003 dB at the 30 dB setting.

S7.8 **Resolution of the reference attenuator setting (δL_{ia}, δL_{ib}):** The digital readout of the reference attenuator has a resolution of 0,001 dB from which the correction for resolution is estimated to be within ±0,0005 dB.

S7.9 **Resolution of the null detector (δL_{0a}, δL_{0b}):** The detector resolution was determined from a previous evaluation to have a standard deviation of 0,002 dB at each reading with assumed normal probability distribution.

S7.10 **Correlation:** None of the input quantities are considered to be correlated to any significant extent.

S7.11 Measurements (L_S): Four observations are made of the incremental loss of the attenuator to be calibrated between settings of 0 dB and 30 dB:

obs. no.	**obs. values at**	
	0 dB setting	30 dB setting
1	0,000 dB	30,033 dB
2	0,000 dB	30,058 dB
3	0,000 dB	30,018 dB
4	0,000 dB	30,052 dB

arithmetic mean: $\bar{L}_S = 30{,}040\ \mathrm{dB}$

experimental standard deviation: $s(L_S) = 0{,}018$ dB

standard uncertainty: $u(L_S) = s(\bar{L}_S) = \dfrac{0{,}018\ \mathrm{dB}}{\sqrt{4}} = 0{,}009\ \mathrm{dB}$

S7.12 Uncertainty budget (L_X):

quantity X_i	estimate x_i	standard uncertainty $u(x_i)$	probability distribution	sensitivity coefficient c_i	uncertainty contribution $u_i(y)$
L_S	30,040 dB	0,0090 dB	normal	1,0	0,0090 dB
δL_S	0,003 dB	0,0025 dB	rectangular	1,0	0,0025 dB
δL_D	0 dB	0,0011 dB	U-shaped	1,0	0,0011 dB
δL_M	0 dB	0,0200 dB	U-shaped	1,0	0,0200 dB
δL_K	0 dB	0,0017 dB	U-shaped	1,0	0,0017 dB
δL_{ia}	0 dB	0,0003 dB	U-shaped	-1,0	-0,0003 dB
δL_{ib}	0 dB	0,0003 dB	rectangular	1,0	0,0019 dB
δL_{0a}	0 dB	0,0020 dB	rectangular	-1,0	0,0020 dB
δL_{0b}	0 dB	0,0020 dB	normal	1,0	-0,0020 dB
L_X	30,043 dB				0,0224 dB

S7.13 Expanded uncertainty:

$U = k \times u(L_X) = 2 \times 0{,}0224\ \mathrm{dB} \cong 0{,}045\ \mathrm{dB}$

S7.14 Reported result:

The measured value of the step attenuator for a setting of 30 dB at 10 GHz is (30,043 ±0,045) dB.

The reported expanded uncertainty of measurement is stated as the standard uncertainty of measurement multiplied by the coverage factor k = 2, which for a normal distribution corresponds to a coverage probability of approximately 95 %.

S7.15 Reference

[1] Harris, I. A. ; Warner, F. L. : *Re-examination of mismatch uncertainty when measuring microwave power and attenuation*. In: IEE Proc., Vol. 128, Pt. H, No. 1, Febr. 1981

SUPPLEMENT 2

Examples

CONTENTS

S8 INTRODUCTION

S8.1 The following examples are chosen to demonstrate further the method of evaluating the uncertainty of measurement. They supplement the examples presented in Supplement 1 to EAL-R2 (Edition 1, November 1997). The present collection of examples focuses on situations where there are one or two dominant terms in the uncertainty propagation or where the number of repeated measurements is small.

S8.2 The examples are chosen to illustrate situations encountered in practice. It should be emphasised, however, that in practical applications there is no need to go through the mathematical derivations presented in these examples, in particular in the mathematical notes appended to some of the examples. Rather, the user is encouraged to employ the results of the theoretical presentations after having made himself acquainted with the conditions that have to be fulfilled. For instance, if it is ascertained, in a given situation, that the result of measurement has a rectangular distribution (as would be the case if there were only one term, rectangularly distributed, that needed to be considered in the propagation), one can immediately draw the conclusion that the coverage factor to be used to arrive at a coverage probability of 95 % is k = 1,65 (see S9.14).

S8.3 One general conclusion that may be drawn from the uncertainty propagation is that in the case of only one dominant contribution the type of distribution of this contribution applies for the result of measurement as well. However, to evaluate the uncertainty of the result of measurement, the applicable sensitivity coefficient has to be employed, as usual.

S8.4 It should be added that the situation where there is only one or a few dominant terms to the uncertainty of measurement is often met in connection with less complicated measuring instruments, where the dominant term often is due to the limited resolution of the instrument. Thus it may appear a paradox that the treatment of uncertainty of measurement for less complicated instruments, as shown by the examples of this Supplement, is more complicated than the treatment of the more straight-forward examples in Supplement 1. However, it should be kept in mind that the mathematical derivations, which may be felt as complications, are inserted for pedagogical reasons at places where they are needed instead of presenting them in the main document.

S8.5 The examples are based on drafts prepared by EA Expert Groups. These drafts have been simplified and harmonised to make them transparent to the laboratory staff in all fields of calibration. It is thus hoped that this set of examples, like the preceding set published as Supplement 1 to EAL-R2, will contribute to a better understanding of the details of setting up the model of evaluation and to the harmonisation of the process of evaluating the uncertainty of measurement, independent of the field of calibration.

S8.6 The contributions and values given in the examples are not intended to imply mandatory or preferred requirements. Laboratories should determine the uncertainty contributions on the basis of the model function they use in the evaluation of the particular calibration they perform and report the evaluated uncertainty of measurement on the calibration certificate they issue.

S8.7 The presentation of the examples follows the common scheme presented and implemented in the first supplement to EAL-R2. For details the reader is referred to clause S1.4 of that document.

S8.8 The uncertainty analysis of the examples is intended to represent the fundamentals of the specific measurement process and the method of evaluating the measurement result and the associated uncertainty. To keep the analysis transparent, also for those who are not experts in the relevant metrological field, a uniform method for the choice of the symbols of quantities has been followed, focused more on the physical background than on the current practice in different fields.

S8.9 There are several recurrent quantities involved in all cases. One of them is the measurand, i.e. the quantity to be measured, another is the quantity presented by the working standard, which realises the local unit; with this quantity the measurand is compared. Besides these two quantities there are several others, in all cases, which take the role of additional local quantities or corrections.

S8.10 Corrections describe the imperfect equality between a measurand and the result of a measurement. Some of the corrections are given by complete results of measurement, i.e. a measured value and its associated measurement uncertainty. For others the distribution of values is inferred from more or less complete knowledge of their nature. In most cases this will lead to an estimation of the limits for the unknown deviations.

S8.11 In certain cases the quantity presented by a working standard is characterised by the nominal value of the standard. Thus nominal values, which generally speaking characterise or identify calibration artefacts, often enter the uncertainty analysis.

S8.12 To distinguish in the mathematical models of evaluation between these concepts, the examples have been designed to follow the notational rules given below. It is evident, however, that it is not possible to follow such rules strictly, because the practice concerning the use of symbols is different in different metrological fields.

S8.13 The notation applied here distinguishes between main values, nominal values, correction values and values of limits:

Main values are measured or observed values that contribute an essential part to the value of a measurand. They are represented by lower-case letters in italics; they will be preceded by an upper-case Greek delta if the quantity represents a difference.

EXAMPLE:

t_{iX} - temperature indicated by a thermometer X to be calibrated. (index i means indicated),

Δl - observed difference in the displacement of a measuring spindle.

Nominal values are assigned values of the realisation of a quantity by a standard or a measuring instrument. They are approximate values that give the main part of the realised value. They are represented by upper-case letters in italics.

EXAMPLE:

L - nominal length of a gauge block to be calibrated.

Correction values give small deviations from the main values that are known or have to be estimated. In most cases they are additive. They are represented by the symbol chosen for the quantity under consideration, preceded by a lower-case Greek delta.

EXAMPLE:

δm_{D} - possible deviation because of the drift of the value of a reference weight since its last calibration

δm_{C} - correction for eccentricity of load and magnetic effects in the calibration of a weight.

Values of limits are fixed, estimated values of possible variations of the unknown values of a quantity. They are represented by the symbol chosen for the quantity under consideration, preceded by a upper-case Greek delta.

EXAMPLE:

$\Delta\alpha_{\mathrm{X}}$ - estimated half-width of the interval of possible deviations of a linear thermal resistivity coefficient given in a manufacturer's specification for a resistor to be calibrated.

The differentiation between different quantities of the same kind is effected by indices as shown in the examples. The internationally accepted notational rules for physical quantities have been followed: indices representing physical quantities are given in italics whereas indices that symbolise artefacts, instruments and so on are written in upright letters.

S8.14 Defined reference values are represented by a quantity symbol with the index zero.

EXAMPLE:

p_0 - reference pressure, e.g. of 1000 mbar.

S8.15 Ratios of quantities of the same kind (dimensionless ratios) are represented by lower-case letters in italics.

EXAMPLE:

$r = R_{\mathrm{iX}} / R_{\mathrm{iN}}$ - ratio of indicated resistance of an unknown resistor and a reference resistor (index *i* means indicated).

S8.16 If several indices are used, the sequence of indices is chosen in such a way that the index representing the most general concept is leftmost and the one representing the most specific concept is rightmost.

EXAMPLE:

$V_{\mathrm{i}1}, V_{\mathrm{i}2}$ - voltage indicated by voltmeter '1' and voltmeter '2', respectively

S8.17 The examples in this second supplement to EAL-R2 are intended to be followed by others, illustrating different aspects encountered in connection with the calibration of measuring instruments. Examples may also be found in EAL and EA Guidance Documents[1] dealing with the calibration of specific types of measuring instruments.

S9 CALBRATION OF A HAND-HELD DIGITAL MULTIMETER AT 100 V DC

S9.1 As part of a general calibration, a hand-held digital multimeter (DMM) is calibrated at an input of 100 V DC using a multifunction calibrator as a working standard. The following measuring procedure is used:

(1) The calibrator's output terminals are connected to the input terminals of the DMM using suitable measuring wires.

(2) The calibrator is set to its 100V setting and, after a suitable stabilising period, the DMM reading is noted.

(3) The error of indication of the DMM is calculated using the DMM readings and the calibrator settings.

S9.2 It must be noted that the error of indication of the DMM which is obtained using this measuring procedure includes the effect of offset as well as deviations from linearity.

S9.3 The error of indication E_{X} of the DMM to be calibrated is obtained from

$$E_{\mathrm{X}} = V_{\mathrm{iX}} - V_{\mathrm{S}} + \delta V_{\mathrm{iX}} - \delta V_{\mathrm{S}} \qquad \text{(S9.1)}$$

where

V_{iX} - voltage, indicated by the DMM (index i means indication),

V_{S} - voltage generated by the calibrator,

δV_{iX} - correction of the indicated voltage due to the finite resolution of the DMM,

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[1] EAL-G26, Calibration of pressure balances
EAL-G31, Calibration of thermocouples
EAL-G32, Measurement and generation of small ac voltages with inductive voltage dividers
EA-10/10, EA Guidelines on the Determination of Pitch Diameter of Parallel Thread gauges by Mechanical Probing

δV_S - correction of the calibrator voltage due to
(1) drift since its last calibration,
(2) deviations resulting from the combined effect of offset, non-linearity and differences in gain,
(3) deviations in the ambient temperature,
(4) deviations in mains power,
(5) loading effects resulting from the finite input resistance of the DMM to be calibrated.

S9.4 Because of the limited resolution of the indication of the DMM, no scatter in the indicated values is observed.

S9.5 DMM readings (V_{ix})

The DMM indicates the voltage 100,1 V at the calibrator setting 100 V. The DMM reading is assumed to be exact (see S9.4).

S9.6 Working standard (V_S)

The calibration certificate for the multifunction calibrator states that the voltage generated is the value indicated by the calibrator setting and that the associated expanded relative uncertainty of measurement is W=0,000 02 (coverage factor $k = 2$), resulting in an expanded uncertainty of measurement associated with the 100 V setting of U=0,002 V (coverage factor $k = 2$).

S9.7 Resolution of DMM to be calibrated (δV_{ix})

The least significant digit of the DMM display corresponds to 0,1 V. Each DMM reading has a correction due to the finite resolution of the display which is estimated to be 0,0 V with limits of $\pm$0.05 V (i.e. one half of the magnitude of the least significant digit).

S9.8 Other corrections (δV_S)

Because of the fact that individual figures are not available the uncertainty of measurement associated with the different sources is derived from the accuracy specification given by the manufacturer of the calibrator. These specifications state that the voltage generated by the calibrator coincides with the calibrator setting within $\pm(0{,}000\,1\times V_S + 1\text{ mV})$[2] under the measuring conditions

(1) the ambient temperature is within the range 18 °C to 23 °C
(2) the mains voltage powering the calibrator is in the range 210 V to 250 V,
(3) the resistive load at the calibrator's terminals is greater than 100 kΩ,
(4) the calibrator has been calibrated within the last year.

Since these conditions of measurement are fulfilled and the calibration history of the calibrator shows that the manufacturer's specification may be relied upon, the correction to be applied to the voltage generated by the calibrator is assumed to be 0,0 V within ±0,011 V.

S9.9 Correlation

[2] A widely used method of presenting accuracy specification of measuring instruments in data sheets or manuals consists in giving the specification limits in terms of 'settings'. For the calibrator, the statement would be ±(0,01% of setting + 1 mV). Even if this method is considered to be equivalent to the expression given above it is not used here because it may be misleading in many cases and because it does not represent an equation of physical quantities in the internationally accepted symbolic nomenclature.

None of the input quantities are considered to be correlated to any significant extent.

S9.10 Uncertainty budget (E_X)

quantity X_i	estimate x_i	standard uncertainty $u(x_i)$	probability distribution	sensitivity coefficient c_i	uncertainty contribution $u_i(y)$
V_{iX}	100,1 V	-	-	-	-
V_S	100,0 V	0,001 V	normal	-1,0	-0,001 V
δV_{iX}	0,0 V	0,029 V	rectangular	1,0	0,029 V
δV_S	0,0 V	0,0064 V	rectangular	-1,0	-0,0064 V
E_X	0,1 V				0,030 V

S9.11 Expanded uncertainty

The standard uncertainty of measurement associated with the result is clearly dominated by the effect of the finite resolution of the DMM. The final distribution is not normal but essentially rectangular. Therefore, the method of effective degrees of freedom described in Annex E of EAL-R2 is not applicable. The coverage factor appropriate for a rectangular distribution is calculated from the relation given in eq. (S9.8) in the mathematical note S9.14.

$$U = k \cdot u(E_X) = 1{,}65 \cdot 0{,}030 \text{ V} \cong 0{,}05 \text{ V}$$

S9.12 Reported result

The measured error of indication of the hand-held digital voltmeter at 100 V is (0,10 ±0,05) V.

The reported expanded uncertainty of measurement is stated as the standard uncertainty of measurement multiplied by the coverage factor $k = 1{,}65$ which has been derived from the assumed rectangular probability distribution for a coverage probability of 95%.

S9.13 Additional remark

The method used for calculating the coverage factor is clearly related to the fact that the measurement uncertainty associated with the result is dominated by the effect of the finite resolution of the DMM. This will be true for the calibration of all low-resolution indicating instruments provided the finite resolution is the only dominant source in the uncertainty budget.

S9.14 Mathematical note

If the situation of measurement is such that one of the uncertainty contributions in the budget can be identified as a dominant term, for instance the term with index 1, the standard uncertainty to be associated with the measurement result y can be written as

$$u(y) = \sqrt{u_1^2(y) + u_\mathrm{R}^2(y)}\,. \tag{S9.2}$$

Here is

$$u_\mathrm{R}(y) = \sqrt{\sum_{i=2}^{N} u_i^2(y)} \tag{S9.3}$$

denotes the total uncertainty contribution of the non-dominant terms. As long as the ratio of the total uncertainty contribution $u_\mathrm{R}(y)$ of the non-dominant terms to the uncertainty contribution $u_1(y)$ of the dominant term is not larger than 0,3, eq. (S9.2) may be approximated by

$$u(y) \cong u_1(y) \cdot \left[1 + \frac{1}{2}\left(\frac{u_\mathrm{R}(y)}{u_1(y)}\right)^2\right]. \tag{S9.4}$$

The relative error of approximation is smaller than 1×10^{-3}. The maximum relative change in the standard uncertainty resulting from the factor within the brackets in eq. (S9.4) is not larger than 5%. This value is within the accepted tolerance for mathematical rounding of uncertainty values.

Under these assumptions the distribution of values that could reasonably be attributed to the measurand is essentially identical with the distribution resulting from the known dominant contribution. From this distribution density $\varphi(y)$ the coverage probability p may be determined for any value of the expanded measurement uncertainty U by the integral relation

$$p(U) = \int_{y-U}^{y+U} \varphi(y')dy'\,. \tag{S9.5}$$

Inverting this relation for a given coverage probability results in the relation between the expanded measurement uncertainty and the coverage probability $U = U(p)$ for the given distribution density $\varphi(y)$. Using this relation, the coverage factor may finally be expressed as

$$k(p) = \frac{U(p)}{u(y)}\,. \tag{S9.6}$$

In the case of the hand-held digital voltmeter the dominant uncertainty contribution resulting from the finite resolution of the indication is $u_{\delta V_X}(E_X) = 0{,}029\ \mathrm{V}$ whereas the total uncertainty contribution of the non-dominant terms is $u_R(E_X) = 0{,}0064\ \mathrm{V}$. The relevant ratio is $u_R(E_X)/u_{\delta V_X}(E_X) = 0{,}22$. Thus the resulting distribution of values that can reasonably be attributed as errors of indications is essentially rectangular. The coverage probability for a rectangular distribution is linearly related to the expanded measurement uncertainty (a being the half-width of the rectangular distribution)

$$p = \frac{U}{a}\,. \qquad \text{(S9.7)}$$

Solving this relation for the expanded measurement uncertainty U and inserting the result together with the expression of the standard measurement uncertainty related to a rectangular distribution as given by eq. (3.8) of EAL-R2 finally gives the relation

$$k(p) = p\sqrt{3}\ . \qquad \text{(S9.8)}$$

For a coverage probability $p = 95\,\%$ applicable in the EA, the relevant coverage factor is thus $k = 1{,}65$.

S10 CALIBRATION OF A VERNIER CALLIPER

S10.1 A vernier calliper made of steel is calibrated against grade I gauge blocks of steel used as working standards. The measurement range of the calliper is 150 mm. The reading interval of the calliper is 0,05 mm (the main scale interval is 1 mm and the vernier scale interval 1/20 mm). Several gauge blocks with nominal lengths in the range 0,5 - 150 mm are used in the calibration. They are selected in such a way that the measurement points are spaced at nearly equal distances (e.g. at 0 mm, 50 mm, 100 mm, 150 mm) but give different values on the vernier scale (e.g. 0,0 mm, 0,3 mm, 0,6 mm, 0,9 mm). The example concerns the 150 mm calibration point for measurement of external dimensions. Before calibration several checks of the condition of the calliper are made. These include dependence of the result of measurement on the distance of the measured item from the beam (Abbe error), quality of the measuring faces of the jaws (flatness, parallelism, squareness), and function of the locking mechanism.

S10.2 The error of indication E_X of the calliper at the reference temperature $t_0 = 20°\mathrm{C}$ is obtained from the relation:

$$E_X = l_{iX} - l_S + L_S \cdot \bar{\alpha} \cdot \Delta t + \delta l_{iX} + \delta l_M \qquad \text{(S10.1)}$$

where:

l_{iX}	-	indication of the calliper,
l_S	-	length of the actual gauge block,
L_S	-	nominal length of the actual gauge block,
$\overline{\alpha}$	-	average thermal expansion coefficient of the calliper and the gauge block,
Δt	-	difference in temperature between the calliper and the gauge block,
δl_{iX}	-	correction due to the finite resolution of the calliper,
δl_M	-	correction due to mechanical effects, such as applied measurement force, Abbe errors, flatness and parallelism errors of the measurement faces.

S10.3 Working standards (l_S, L_S)

The lengths of the reference gauge blocks used as working standards, together with their associated expanded uncertainty of measurement, are given in the calibration certificate. This certificate confirms that the gauge blocks comply with the requirements for grade I gauge blocks according to ISO 3650, i.e. that the central length of the gauge blocks coincides within ±0,8 µm with the nominal length. For the actual lengths of the gauge blocks their nominal lengths are used without correction, taking the tolerance limits as the upper and lower limits of the interval of variability.

S10.4 Temperature (Δt, $\overline{\alpha}$)

After an adequate stabilisation time, the temperatures of the calliper and the gauge block are equal within ±2 °C. The average thermal expansion coefficient is $11{,}5 \cdot 10^{-6}$ $°C^{-1}$. (The uncertainty in the average thermal expansion coefficient and in the difference of the thermal expansion coefficients has not been taken into account; its influence is considered negligible for the present case. Cf. EAL-R2-S1, example S4.)

S10.5 Resolution of the calliper (δl_{iX})

The scale interval of the vernier scale is 0,05 mm. Thus variations due to the finite resolution are estimated to have rectangular limits of ± 25 µm.

S10.6 Mechanical effects (δl_M)

These effects include the applied measurement force, the Abbe error and the play between the beam and the sliding jaw. Additional effects may be caused by the fact that the measuring faces of the jaws are not exactly flat, not parallel to each other and not perpendicular to the beam. To minimise effort, only the range of the total variation, equal to ±50 µm is considered.

S10.7 Correlation

None of the input quantities are considered to be correlated to any significant extent.

S10.8 Measurements (l_{iX})

The measurement is repeated several times without detecting any scatter in the observations. Thus uncertainty due to limited repeatability does not give a contribution. The result of measurement for the 150 mm gauge block is 150,10 mm.

S10.9 Uncertainty budget (δl_X)

quantity X_i	estimate x_i	standard uncertainty $u(x_i)$	probability distribution	sensitivity coefficient c_i	uncertainty contribution $u_i(y)$
l_{iX}	150,10 mm	-	-	-	-
l_S	150,00 m	0,46 µm	rectangular	-1,0	-0,46 µm
Δt	0	1,15 K	rectangular	1,7 µMk^{-1}	2,0 µm
δl_{iX}	0	15 µm	rectangular	1,0	15 µm
δl_M	0	29 µm	rectangular	1,0	29 µm
E_X	0,10 mm				33 µm

S10.10 Expanded uncertainty

The uncertainty of measurement associated with the result is clearly dominated by the combined effect of the measurement force and the finite resolution of the vernier. The final distribution is not normal but essentially trapezoidal with a ratio $\beta = 0{,}33$ of the half-width of the plateau region to the half-width of the variability interval. Therefore the method of effective degrees of freedom described in EAL-R2, Annex E is not applicable. The coverage factor k = 1,83 appropriate for this trapezoidal distribution of values is calculated from eq. (S10.10) of the mathematical note S10.13. Thus

$$U = k \cdot u(E_X) = 1{,}83 \cdot 0{,}033\ \text{mm} \cong 0{,}06\ \text{mm}$$

S10.11 Reported result

At 150 mm the error of indication of the calliper is (0,10 ± 0,06) mm.

The reported expanded uncertainty of measurement is stated as the standard uncertainty of measurement multiplied by the coverage factor $k = 1{,}83$ which has been derived from the assumed trapezoidal probability distribution for a coverage probability of 95 %.

S10.12 Additional remark

The method used for calculating the coverage factor is clearly related to the fact that uncertainty of measurement associated with the result is dominated by two influences: the mechanical effects and the finite resolution of the vernier scale. Thus the assumption of a normal distribution for the output quantity is not justified and the conditions of EAL-R2, paragraph 5.6 apply. In the sense that probabilities and probability densities in practice may only be determined to within 3 %– 5 %, the distribution is essentially trapezoidal, obtained by convolution of the two rectangular distributions associated with the dominant contributions. The half-widths of the base and the top of the resulting symmetrical trapezoid are 75 µm and 25 µm, respectively. 95 % of the area of the trapezoid is encompassed by an interval ±60 µm around its symmetry axis, corresponding to $k = 1{,}83$.

S10.13 Mathematical note

If the situation of measurement is such that two of the uncertainty contributions in the budget can be identified as dominant terms, the method presented in S9.14 can be applied when the two dominant contributions, for instance the terms with indices

1 and 2 , are combined into one dominant term. The standard uncertainty to be associated with the measurement result *y* may be written in this case as

$$u(y) = \sqrt{u_0^2(y) + u_\mathrm{R}^2(y)} \tag{S10.2}$$

where

$$u_0(y) = \sqrt{u_1^2(y) + u_2^2(y)} \tag{S10.3}$$

denotes the combined contribution of the two dominant terms and

$$u_\mathrm{R}(y) = \sqrt{\sum_{i=3}^{N} u_i^2(y)} \tag{S10.4}$$

the total uncertainty contribution of the remaining non-dominant terms. If the two dominant contributions arise from rectangular distributions of values with half-widths a_1 and a_2, the distribution resulting from convolving them is a symmetrical trapezoidal distribution

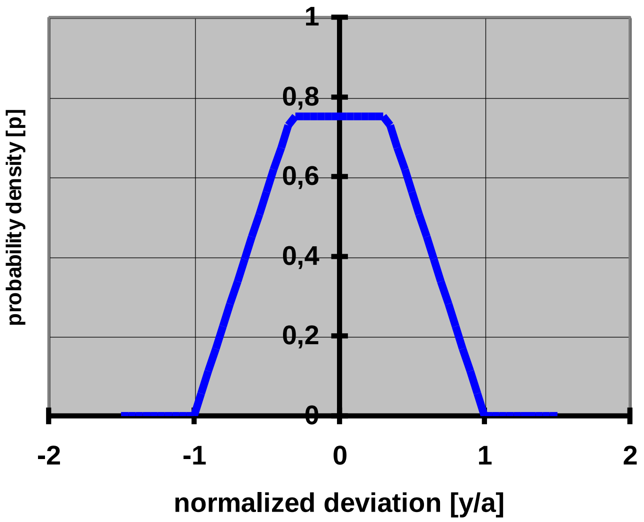

Fig. 1: Unified symmetrical trapezoidal probability distribution with the value β=0,33 of the edge parameter, resulting from the convolution of two rectangular distributions.

with half-widths

$$a = a_1 + a_2 \text{ and } b = |a_1 - a_2| \qquad \text{(S10.5)}$$

of the base and the top, respectively (see example in Fig. 1). The distribution may be conveniently expressed in the unified form

$$\varphi(y) = \frac{1}{a(1+\beta)} \times \begin{cases} 1 & |y| < \beta \cdot a \\ \frac{1}{1-\beta}\left(1 - \frac{|y|}{a}\right) & \beta \cdot a \le |y| \le a \\ 0 & a < |y| \end{cases} \qquad \text{(S10.6)}$$

with the edge parameter

$$\beta = \frac{b}{a} = \frac{|a_1 - a_2|}{a_1 + a_2} \qquad \text{(S10.7)}$$

The square of the standard measurement uncertainty deduced from the trapezoidal distribution of eq. (S10.6) is

$$u^2(y) = \frac{a^2}{6}(1+\beta^2) \ . \qquad \text{(S10.8)}$$

Using the distribution of eq. (S10.6) the dependence of the coverage factor on the coverage probability is derived according to the method sketched in S9.14

$$k(p) = \frac{1}{\sqrt{\frac{1+\beta^2}{6}}} \times \begin{cases} \frac{p(1+\beta)}{2} & \frac{p}{2-p} < \beta \\ 1 - \sqrt{(1-p)(1-\beta^2)} & \beta \le \frac{p}{2-p} \end{cases} . \qquad \text{(S10.9)}$$

Fig. 2 shows the dependence of the coverage factor k on the value of the edge parameter β for a coverage probability of 95 %.

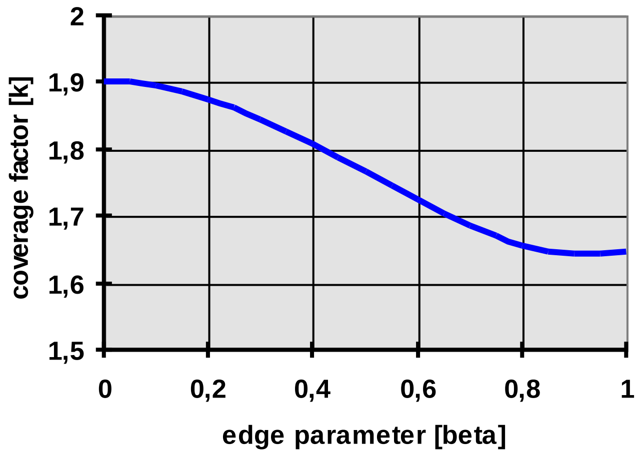

Fig. 2: Dependence of the coverage factor *k* on the value of the edge parameter β of a trapezoidal distribution for a coverage probability of 95 %.

The coverage factor for a coverage probability of 95 % appropriate to a trapezoidal distribution with an edge parameter of $\beta < 0{,}95$ is calculated from the relation

$$k = \frac{1-\sqrt{(1-p)(1-\beta^2)}}{\sqrt{\frac{1+\beta^2}{6}}} \; . \qquad \text{(S10.10)}$$

S11 CALIBRATION OF A TEMPERATURE BLOCK CALIBRATOR AT A TEMPERATURE OF 180°C[3]

S11.1 As part of a calibration, the temperature that has to be assigned to the calibration bore of a temperature block calibrator, is measured. This is done when the indication of the built-in temperature indicator has stabilised at 180,0 °C. The temperature of the calibration bore is determined by an inserted platinum resistance thermometer, used as a working standard, by measuring the electrical resistance of the thermometer by an ac resistance bridge. The temperature t_X, that has to be assigned as the temperature of the bore when the reading of the built-in temperature indicator is 180,0 °C, is given by:

$$t_X = t_S + \delta t_S + \delta t_D - \delta t_{iX} + \delta t_R + \delta t_A + \delta t_H + \delta t_V \qquad \text{(S11.1)}$$

[3] A similar example will be found in the EA guideline EA-10/xx, Calibration of temperature block calibrators. It has been included here, in a dimplified form, in order to highligt how a value is assigned to an indication of an instrument in a calibration process. This process is basic for calibrations in different metrological fields and, therefore, of general interest. The example further demonstrates that there are two equivalent ways to tackle this problem: the direct assignment of a value to the indication of the instrument and the association of a correction to the indication, usually called the error of indication.

where:

t_S	-	temperature of the working standard derived from the ac resistance measurement,
δt_S	-	temperature correction due to the ac resistance measurement,
δt_D	-	temperature correction due to drift in the value of the working standard since its last calibration,
δt_{iX}	-	temperature correction due to the settability limitations of the block temperature calibrator,
δt_R	-	temperature correction due to the radial temperature difference between the built-in thermometer and the working standard,
δt_A	-	temperature correction due to the axial inhomogeneity of temperature in the measuring bore,
δt_H	-	temperature correction due to hysteresis in the increasing and decreasing branches of the measuring cycle,
δt_V	-	temperature variation within the time of measurement.

Temperature corrections due to stem conduction are not considered, since the platinum resistance thermometer used as working standard has an outer diameter $d \leq 6\ \mathrm{mm}$. Prior investigations have shown that stem conduction effects can be neglected in this case.

S11.2 Working standard (t_S)

The calibration certificate of the resistance thermometer used as working standard gives the relationship between resistance and temperature. The measured resistance value corresponds to a temperature of 180,1 °C, with an associated expanded uncertainty of measurement *U* = 30 mK (coverage factor $k = 2$).

S11.3 Determination of the temperature by resistance measurement(δt_S)

The temperature of the resistance thermometer used as working standard is determined as 180,1 °C. The standard measurement uncertainty associated with the resistance measurement converted to temperature corresponds to $u(\delta t_S) = 10\ \mathrm{mK}$.

S11.4 Drift of the temperature of the working standard (δt_D)

From general experience with platinum resistance thermometers of the type used as working standard in the measurement, the change of temperature due to resistance ageing since the last calibration of the standard is estimated to be within the limits ±40 mK.

S11.5 Settability of the block temperature calibrator (δt_{iX})

The built-in controlling thermometer of the block temperature calibrator has a scale interval of 0,1 K. This gives temperature resolution limits of ±50 mK within which the thermodynamic state of the temperature block can be uniquely set.

Note: If the indication of the built-in temperature indicator is not given in units of temperature the resolution limits must be converted into equivalent temperature values by multiplying the indication with the relevant instrument constant.

S11.6 Radial inhomogeneity of temperature (δt_{R})

The radial temperature difference between the measuring bore and the built-in thermometer has been estimated to be within ±100 mK.

S11.7 Axial inhomogeneity of temperature (δt_{A})

The temperature deviations due to axial inhomogeneity of temperature in the calibration bore have been estimated from readings for different immersion depths to be within ±250 mK.

S11.8 Hysteresis effects (δt_{H})

From readings of the reference thermometer during measurement cycles of increasing and decreasing temperature, the temperature deviation of the calibration bore due to hysteresis effect has been estimated to be within ±50 mK.

S11.9 Temperature instability (δt_{V})

Temperature variations due to temperature instability during the measuring cycle of 30 min are estimated to be within ±30 mK.

S11.10 Correlations

None of the input quantities are considered to be correlated to any significant extent.

S11.11 Repeated observations

Due to the finite resolution of the indication of the built-in thermometer no scatter in the indicated values has been observed and taken into account.

S11.12 Uncertainty budget (t_{X})

Quantity X_i	Estimate x_i	Standard uncertainty $u(x_i)$	Probability distribution	Sensitivity coefficient c_i	Uncertainty contribution $u_i(y)$
t_{S}	180,1 °C	15 mK	normal	1,0	15 mK
δt_{S}	0,0 °C	10 mK	normal	1,0	10 mK
δt_{D}	0,0 °C	23 mK	rectangular	1,0	23 mK
δt_{iX}	0,0 °C	29 mK	rectangular	-1,0	-29 mK
δt_{R}	0,0 °C	58 mK	rectangular	1,0	58 mK
δt_{A}	0,0 °C	144 mK	rectangular	1,0	144 mK
δt_{H}	0,0 °C	29 mK	rectangular	1,0	29 mK
δt_{V}	0,0 °C	17 mK	rectangular	1,0	17 mK
t_{X}	180,1 °C				164 mK

S11.13 Expanded uncertainty

The standard uncertainty of measurement associated with the result is clearly dominated by the effect of the unknown temperature correction due to the axial temperature inhomogeneity in the measuring bore and the radial temperature difference between the built-in thermometer and the working standard. The final distribution is not normal but essentially trapezoidal. According to S10.13, the coverage factor corresponding to the edge parameter $\beta = 0{,}43$ is $k = 1{,}81$.

$$U = k \cdot u(t_{\mathrm{X}}) = 1{,}81 \cdot 164\ \mathrm{mK} \cong 0{,}3\ \mathrm{K}$$

S11.14 Reported result

The temperature of the calibration bore that has to be assigned to an indication of the built-in controlling thermometer of 180,0 °C is 180,1 °C ± 0,3 °C.

The reported expanded uncertainty of measurement is stated as the standard uncertainty of measurement multiplied by the coverage factor $k = 1{,}81$ which has been derived from the assumed trapezoidal probability distribution for a coverage probability of 95 %.

S11.15 Mathematical note concerning the model

Some metrologists are confused that the indication of the controlling thermometer does not appear explicitly in the model function of eq. (S11.1). To fit their needs, the problem can alternatively be formulated with the error of indication

$$E_{\mathrm{X}} = t_{\mathrm{X}} - t_{\mathrm{i}} \tag{S11.2}$$

of the built-in temperature indicator

$$E_{\mathrm{X}} = t_{\mathrm{S}} - t_{\mathrm{i}} + \delta t_{\mathrm{S}} + \delta t_{\mathrm{D}} - \delta t_{\mathrm{iX}} + \delta t_{\mathrm{R}} + \delta t_{\mathrm{A}} + \delta t_{\mathrm{H}} + \delta t_{\mathrm{V}} \tag{S11.3}$$

The indicated value t_{i} is a nominal value. Its effect is to shift the scale of the measurand. It does, however, not contribute to the uncertainty of measurement associated with the error of indication

$$u(E_{\mathrm{X}}) = u(t_{\mathrm{X}}) \tag{S11.4}$$

The model function of eq. (S11.1) can be regained from eq. (S11.3) using the definition of the error of indication in eq. (S11.2).

This note shows that there is not necessarily only one unique way to choose the model of evaluation of measurement. The metrologist keeps it in his hands to choose the model that suits his habits and his approach to the problem. Model functions that can be transformed mathematically from one into the other represent the same measurement process. For cases where a continuous scale of indication is involved, as in the calibration of the temperature block under consideration, model functions that are connected by linear scale transformations may serve as equivalent expressions of the measurement problem.

S12 CALIBRATION OF A HOUSEHOLD WATER METER

S12.1 The calibration of a water meter involves the determination of the relative error of indication within the applicable flow range of the meter. The measurement is made using a test rig that supplies necessary water flow with a pressure of approximately 500 kPa, a value typical for municipal tap water systems. The water is received in an open collecting tank that has been calibrated and determines the reference volume of the water. It is empty but wetted at the beginning of the measurement. The collecting tank has a narrow neck with an attached scale by which the filling level can be detected. The meter to be calibrated is connected between these tanks. It has a mechanical counter with pointers. The measurement is done at a flow rate of 2500 l/h with standing start-and-stop which means that the flow rate is zero both at the beginning and the end of the measurement. The indication of the meter is recorded at the beginning and at the end of the measurement. The level is recorded in the collecting tank at the end of the measurement. The temperature and pressure of the water at the meter, and the temperature of the water in the collecting tank, are recorded as well.

S12.2 The relative error of indication e_X in a single run is defined as

$$e_\text{X} = \frac{\Delta V_\text{iX} + \delta V_\text{iX2} - \delta V_\text{iX1}}{V_\text{X}} - 1 \qquad \text{(S12.1)}$$

with

$$V_\text{X} = (V_\text{iS} + \delta V_\text{iS})(1 + \alpha_\text{S}(t_\text{S} - t_0))(1 + \alpha_\text{W}(t_\text{X} - t_\text{S}))(1 - \kappa_\text{W}(p_\text{X} - p_\text{S})) \qquad \text{(S12.2)}$$

where:

$\Delta V_\text{iX} = V_\text{iX2} - V_\text{iX1}$	-	difference in meter indications,
V_iX1, V_iX2	-	indication of the meter at the beginning of the measurement and at the end of the measurement,
$\delta V_\text{iX1}, \delta V_\text{iX2}$	-	corrections due to the finite resolution of the meter indication,
V_X	-	volume that passed the meter during the measurement under the prevailing conditions, i.e. pressure p_X and temperature t_X, at the inlet of the meter,
V_iS	-	volume indicated at the neck scale of the collecting tank at the end of the measurement,
δV_iS	-	correction of the volume indicated at the neck scale of the collecting tank due to the finite resolution of the scale,
α_S	-	cubic thermal expansion coefficient of the material of the collecting tank,
t_S	-	temperature of the collecting tank,
t_0	-	reference temperature at which the collecting tank has been calibrated,
α_W	-	cubic thermal expansion coefficient of water,
t_X	-	temperature of the water at the inlet of the meter,
κ_W	-	compressibility of water,
p_S	-	pressure in the collecting tank (it is zero if excess pressure is considered)
p_X	-	pressure of the water at the inlet of the meter.

S12.3 Collecting tank (V_{iS}, t_0)

The calibration certificate states that the neck scale indicates the volume of 200 l at the reference temperature $t_0 = 20\,°\mathrm{C}$ with an associated relative expanded measurement uncertainty of 0,1 % ($k = 2$). The expanded measurement uncertainty associated with the value is 0,2 l ($k = 2$).

S12.4 Resolution of the collecting tank scale (δV_{iS})

The water level of the collecting tank can be determined to within ±1 mm. With the scale factor of the tank of 0,02 l/mm the maximum deviation of the volume of water in the collecting tank from the observed indicated value is estimated to be within ±0,02 l.

S12.5 Temperature of the water and the collecting tank (α_S, t_S)

The temperature of the water in the collecting tank is determined to be 15 °C within ±2 K. The stated limits cover all possible sources of uncertainty, such as calibration of temperature sensors, resolution in reading and temperature gradients in the tank. The cubic thermal expansion coefficient of the tank material (steel) is taken from a material handbook to be a constant equal to $\alpha_S = 51 \cdot 10^{-6}\ \mathrm{K}^{-1}$ in the temperature interval considered. Since there is no uncertainty statement accompanying this value it is assumed to be known to within its least significant digit. Unknown deviations are considered to be within the rounding limits of $\pm 0{,}5 \cdot 10^{-6}\ \mathrm{K}^{-1}$.

S12.6 Temperature of the water at the meter (α_W, t_X)

The temperature of the water at the inlet of the meter is determined to be 16 °C within ±2 K. The stated limits cover all possible sources of uncertainty, such as contributions from calibration of sensors, resolution in reading and small temperature changes during one measurement run. The cubic expansion coefficient of water is taken from a material handbook to be a constant equal to $\alpha_W = 0{,}15 \cdot 10^{-3}\ \mathrm{K}^{-1}$ in the temperature interval considered. Since there is no uncertainty statement accompanying this value it is assumed to be known to within its least significant digit. Unknown deviations are considered to be within the rounding limits of $\pm 0{,}5 \cdot 10^{-6}\ \mathrm{K}^{-1}$.

S12.7 Pressure difference of the water between the meter and the tank (κ_W, p_S, p_X)

The excess pressure of the water supplied to the inlet of the meter is 500 kPa with relative deviations not larger than ±10 %. On its way from the inlet to the collecting tank, the water expands to excess pressure 0 kPa (atmospheric pressure condition). The compressibility of water is taken from a material handbook to be a constant equal to $\kappa_W = 0{,}46 \cdot 10^{-6}\ \mathrm{kPa}^{-1}$ in the temperature interval considered. Since there is no uncertainty statement accompanying this value, it is assumed to be known to within its least significant digit. Unknown deviations are considered to be within the rounding limits of $\pm 0{,}005 \cdot 10^{-6}\ \mathrm{kPa}^{-1}$.

S12.8 Correlation

None of the input quantities are considered to be correlated to any significant extent.

S12.9 Uncertainty budget (V_X)

quantity X_i	estimate x_i	standard uncertainty $u(x_i)$	probability distribution	sensitivity coefficient c_i	uncertainty contribution $u_i(y)$
V_{iS}	200,02 l	0,10 l	normal	1,0	0,10 l
δV_{iS}	0,0 l	0,0115 l	rectangular	1,0	0,0115 l
α_S	$51 \cdot 10^{-6}$ K^{-1}	$0{,}29 \cdot 10^{-6}$ K^{-1}	rectangular	-1000 l·K	$-0{,}29 \cdot 10^{-3}$ l
t_S	15°C	1,15 K	rectangular	-0,0198 l·K^{-1}	-0,0228 l
α_W	$0{,}15 \cdot 10^{-3}$ K^{-1}	$2{,}9 \cdot 10^{-6}$ K^{-1}	rectangular	200 l·K	$0{,}58 \cdot 10^{-3}$ l
t_X	16°C	1,15 K	rectangular	-0,0300 l·K^{-1}	-0,0346 l
κ_W	$0{,}46 \cdot 10^{-6}$ kPa^{-1}	$2{,}9 \cdot 10^{-6}$ kPa^{-1}	rectangular	-100 l·kPa	$-0{,}29 \cdot 10^{-3}$ l
p_X	500 kPa	29 kPa	rectangular	$-9{,}2 \cdot 10^{-6}$ l·kPa^{-1}	-0,0027 l
p_S	0,0 Pa	-	-	-	-
V_X	199,95 l				0,109 l

The standard uncertainty of measurement associated with the result is clearly dominated by the volume indication at the neck scale of the collecting tank. The final distribution is not normal but essentially rectangular. This must be kept in mind in the further processing of the uncertainty evaluation.

S12.10 Indication of the meter (ΔV_{iX}, δV_{iX1}, δV_{iX2})

The water meter to be calibrated has a resolution of 0,2 l resulting in the limits ± 0,1 l in both readings for the maximum deviations resulting from the meter resolution.

S12.11 Uncertainty budget (e_X)

quantity X_i	estimate x_i	standard uncertainty $u(x_i)$	probability distribution	sensitivity coefficient c_i	uncertainty contribution $u_i(y)$
ΔV_{iX}	200,0 l	-	nominal	-	-
δV_{iX1}	0,0 l	0,058 l	rectangular	$-5{,}0 \cdot 10^{-3}$	$-0{,}29 \cdot 10^{-3}$ l
δV_{iX2}	0,0 l	0,058 l	rectangular	$5{,}0 \cdot 10^{-3}$	$0{,}29 \cdot 10^{-3}$ l
V_X	199,95 l	0,109 l	rectangular	$-5{,}0 \cdot 10^{-3}$	$-0{,}55 \cdot 10^{-3}$ l
e_X	0,000 3				$0{,}68 \cdot 10^{-3}$

S12.12 Repeatability of the meter

The relative error of indication of the water meter to be calibrated, determined at the same flow rate of 2500 l/h, shows considerable scatter. For that reason the relative error of indication is determined three times. The results of these three runs are treated as independent observations $e_{\mathrm{X}\,j}$ in the model that determines the average error of indication e_{Xav}:

$$e_{\mathrm{Xav}} = e_{\mathrm{X}} + \delta e_{\mathrm{X}} \qquad \text{(S12.3)}$$

where:

e_{X} - relative error of indication of a single run,

δe_{X} - correction of the relative error of indication obtained in the different runs due to the lack of repeatability of the meter.

S12.13 Measurements (e_{X})

No.	observed relative error of indication
1	0,000 3
2	0,000 5
3	0.002 2

arithmetic mean: $\bar{e}_{\mathrm{X}} = 0{,}001$

experimental standard deviation: $s(e_{\mathrm{X}\,j}) = 0{,}001$

standard uncertainty: $u(\bar{e}_{\mathrm{X}}) = s(\bar{e}_{\mathrm{X}}) = \dfrac{0{,}001}{\sqrt{3}} = 0{,}000\,60$

S12.14 Uncertainty budget (e_{Xav})

quantity X_i	estimate x_i	standard uncertainty $u(x_i)$	degrees of freedom ν_{eff}	probability distribution	sensitivity coefficient c_i	uncertainty contribution $u_i(y)$
e_{X}	0,001	$0{,}60 \cdot 10^{-3}$	2	normal	1,0	$0{,}60 \cdot 10^{-3}$
δe_{X}	0,0	$0{,}68 \cdot 10^{-3}$	∞	normal	1,0	$0{,}68 \cdot 10^{-3}$
e_{Xav}	0,001		10			$0{,}91 \cdot 10^{-3}$

S12.15 Expanded uncertainty

Because of the small number of effective degrees of freedom of the standard uncertainty associated with the mean relative error of indication the standard coverage factor has to be modified according to table E1

$$U = k \cdot u(e_{\mathrm{X\,av}}) = 2{,}28 \cdot 0{,}91 \cdot 10^{-3} \cong 2 \cdot 10^{-3}$$

S12.16 Reported result

The average relative error of indication of the water meter determined at a flow rate of 2500 l/h is 0,001 ± 0,002.

The reported expanded uncertainty of measurement is stated as the standard uncertainty of measurement multiplied by the coverage factor $k = 2{,}28$, which for a t-distribution with $\nu_{\mathrm{eff}} = 10$ effective degrees of freedom corresponds to a coverage probability of approximately 95 %.

S13 CALIBRATION OF A RING GAUGE WITH A NOMINAL DIAMETER OF 90 MM

S13.1 A steel ring gauge of D_x = 90 mm nominal inner diameter is calibrated applying the procedure introduced in EAL-G29. A length comparator of the Abbe type and a steel setting ring, whose nominal inner diameter (D_s = 40 mm) differs significantly from that of the ring to be calibrated, are employed. In this case the length comparator and the steel setting ring both take the role of working standards. The rings are gently clamped sequentially on a 4-degrees of freedom table, which includes all position elements for aligning the test pieces. The rings are contacted at several points diametrically apart by two C-shaped arms, fixed on the steady and the measuring spindle, respectively. The C-shaped arms are supplied with spherical contact tips. The measuring force is generated by a tension weight ensuring a constant force of nominally 1,5 N over the whole measuring range. The measuring spindle is rigidly connected with the gauge head of a steel line scale of resolution 0,1 µm. The line scale of the comparator has been verified periodically to meet the manufacturer's specification of maximum permissible error.

The ambient temperature is monitored in order to maintain the environmental conditions stated by calibration procedure. The temperature in the comparator working volume is maintained at 20 °C within ±0,5 K. Care is taken to ensure that the rings and the line scale (ruler) maintain the monitored temperature throughout the calibration.

S13.2 The diameter d_{X} of the ring to be calibrated at the reference temperature $t_0 = 20$ °C is obtained from the relationship:

$$d_{\mathrm{X}} = d_{\mathrm{S}} + \Delta l + \delta l_{\mathrm{i}} + \delta l_{\mathrm{T}} + \delta l_{\mathrm{P}} + \delta l_{\mathrm{E}} + \delta l_{\mathrm{A}} \qquad \text{(S13.1)}$$

where:

d_S	-	diameter of the reference setting ring at the reference temperature,
Δl	-	observed difference in displacement of the measuring spindle when the contact tips touch the inner surface of the rings at two diametrically apart points,
δl_i	-	correction for the errors of indication of the comparator,
δl_T	-	correction due to the temperature effects of the ring to be calibrated, the reference setting ring and the comparator line scale,
δl_P	-	correction due to coaxial misalignment of the probes with respect to the measuring line,
δl_E	-	correction due to the difference in elastic deformations of the ring to be calibrated and the reference setting ring,
δl_A	-	correction due to the difference of the Abbe errors of the comparator when the diameters of the ring to be calibrated and the reference setting ring are measured.

S13.3 Working standard (d_S)

The inner diameter of the setting ring used as the working standard together with the associated expanded uncertainty of measurement is given in the calibration certificate as 40,0007 mm ± 0,2 µm (coverage factor *k* = 2).

S13.4 Comparator (δl_i)

The corrections for the errors of indication of the line scale (ruler) were determined by the manufacturer and prestored electronically. Any residuals are within the manufacturers specification of $\pm(0{,}3\,\mu\mathrm{m} + 1{,}5\cdot 10^{-6}\cdot l_i)$ with l_i being the indicated length. The specifications are ascertained by periodical verifications. For the actual length difference $D_X - D_S = 50\,\mathrm{mm}$ unknown residuals are estimated to be within ± (0,375) µm.

S13.5 Temperature corrections (δl_T)

Throughout the measurement care is taken to ensure that the ring to be calibrated, the setting ring and the comparator scale maintain the monitored temperature. From previous measurements and general experience with the measurement system it can be ascertained that the deviations of temperatures of the ring to be calibrated, the setting ring and the comparator scale from ambient temperature stay within ±0,2 K. The ambient temperature of the measuring room, however, is estimated to be within ±0,5 K. The knowledge on the measurement, therefore, is best described by the deviation of the ambient temperature from the reference temperature and the deviations of the temperatures of the ring to be calibrated, the setting ring and the comparator scale (ruler) from the ambient temperature. The correction δl_T due to temperature influences is determined from the model:

$$\begin{aligned}\delta l_T = &\left(D_S\cdot(\alpha_S-\alpha_R) - D_X\cdot(\alpha_X-\alpha_X)\right)\cdot\Delta t_A \\ &+ D_S\cdot\alpha_S\cdot\delta t_S - D_X\cdot\alpha_X\cdot\delta t_X - (D_S - D_X)\cdot\alpha_R\cdot\delta t_R\end{aligned} \tag{S13.2}$$

where:

D_X , D_S	-	nominal diameters of the ring to be calibrated and the reference setting ring,
$\alpha_X, \alpha_S, \alpha_R$	-	linear thermal expansion coefficients of the ring to be calibrated, the reference setting ring and the comparator line scale (ruler),
$\Delta t_A = t_A - t_0$	-	deviations of the ambient temperature of the measuring room from the reference temperature $t_0 = 20°\mathrm{C}$,
$\delta t_X, \delta t_S, \delta t_R$	-	deviations of the temperature of the ring to be calibrated, the reference setting ring and the comparator line scale (ruler) from ambient temperature

Since the expectations of the four temperature differences entering eq. (S13.2) are zero, the usual linearized version will not include effects of the measurement uncertainty associated with the values of the three linear thermal expansion coefficients. As depicted in section S4.13 the non-linear version has to be used to determine the standard uncertainty associated with the four product terms:

$$\begin{aligned}
\delta l_{TA} &= \left(D_S \cdot (\alpha_S - \alpha_R) - D_X \cdot (\alpha_X - \alpha_R)\right) \cdot \Delta t_A \\
\delta l_{TS} &= D_S \cdot \alpha_S \cdot \delta t_S \\
\delta l_{TX} &= D_X \cdot \alpha_X \cdot \delta t_X \\
\delta l_{TR} &= (D_S - D_X) \cdot \alpha_R \cdot \delta t_R
\end{aligned} \tag{S13.3}$$

Based on the calibration certificate of the setting ring, on the manufacturer's data for the ring to be calibrated and the comparator scale, the linear thermal expansion coefficients are assumed to be within the interval (11,5 ± 1,0) 10^{-6} $°\mathrm{K}^{-1}$. Using this value and the limits of temperature variation stated at the beginning, the standard uncertainties associated with the four product terms are $u(\delta l_{TA}) = 0{,}012\,\mu\mathrm{m}$, $u(\delta l_{TS}) = 0{,}053\,\mu\mathrm{m}$, $u(\delta l_{TX}) = 0{,}12\,\mu\mathrm{m}$ and $u(\delta l_{TR}) = 0{,}066\,\mu\mathrm{m}$. The standard uncertainty associated with the combined temperature corrections is derived from these values with the use of the following uncertainty sub-budget:

quantity X_i	estimate x_i	standard uncertainty $u(x_i)$	probability distribution	sensitivity coefficient c_i	uncertainty contribution $u_i(y)$
δl_{TA}	0,0 µm	0,012 µm	-	1,0	0,012 µm
δl_{TS}	0,0 µm	0,053 µm	-	1,0	0,053 µm
δl_{TX}	0,0 µm	0,12 µm	-	1,0	0,12 µm
δl_{TR}	0,0 µm	0,066 µm	-	1,0	0,066 µm
δl_T	0,0 µm				0,15 µm

S13.6 Coaxiality correction (δl_{P})

The deviation from coaxiality of the two spherical probes and the measuring line is assumed to be within ±20 µm. Using the equations stated in the mathematical note (S13.13) the correction due to possible non-coaxiality and the associated standard uncertainty of measurement is given by

$$\delta l_{\mathrm{P}} = 2 \cdot \left(\frac{1}{D_{\mathrm{X}}} - \frac{1}{D_{\mathrm{S}}} \right) \cdot u^2(\delta c) \qquad \text{(S13.4)}$$

$$u^2(\delta l_{\mathrm{P}}) = \frac{16}{5} \cdot \left(\frac{1}{D_{\mathrm{X}}^2} + \frac{1}{D_{\mathrm{S}}^2} \right) \cdot u^4(\delta c) \qquad \text{(S13.5)}$$

Here δc is the small distance of the measured cord from the centre of the ring. The values resulting for the correction and the associated standard measurement uncertainty are $\delta l_{\mathrm{P}} \cong -0{,}004\ \mu\mathrm{m}$ and $u(\delta l_{\mathrm{P}}) \cong 0{,}0065\ \mu\mathrm{m}$. As can been seen from the uncertainty budget (S13.10), these values are two orders of magnitude smaller than the remaining uncertainty contributions so that their influence need not be taken into account under the current measurement conditions.

S13.7 Elastic deformation correction (δl_{E})

The elastic deformation of the ring to be calibrated or the reference setting ring are not determined during the current measurement. From previous experience, however, the effects resulting from elastic deformations are estimated to be within ± 0,03 µm.

S13.8 Abbe error correction (δl_{A})

The actual values of the Abbe errors of the comparator are not determined during the current measurement. From experience and periodical verification data of the comparator, however, the effects due to Abbe errors are estimated to be within ± 0,02 µm.

S13.9 Measurements (Δl)

The following observations are made of the inner diameter of the unknown and the setting ring:

No	Object	Observation	Measurand
1	reference setting ring	0 during this step the comparator display is zeroed	diameter in the nominal direction of the symmetry plane orthogonal to the cylinder axis
2	ring to be calibrated	49,99935 mm	diameter in the nominal direction of the symmetry plane orthogonal to the cylinder axis
3	ring to be calibrated	□□,99911 mm	diameter in the symmetry plane orthogonal to the cylinder axis rotated around the axis with respect to the nominal direction by +1 mm on the circumference

No	Object	Observation	Measurand
4	ring to be calibrated	□□,99972 mm	diameter in the symmetry plane orthogonal to the cylinder axis rotated around the axis with respect to the nominal direction by □1 mm on the circumference.
5	ring to be calibrated	□□,99954 mm	diameter in the nominal direction translated to the plane parallel to symmetry plane orthogonal to the cylinder axis by 1 mm upwards
6	ring to be calibrated	□□,99996 mm	diameter in the nominal direction translated to the plane parallel to symmetry plane orthogonal to the cylinder axis by 1 mm downwards

The observations may be divided into two groups: the observation of the diameter of the setting ring (observation no 1) that is used to set the comparator display to zero and the observation of the diameter of the ring to be calibrated (observations no 2 to no 6) that give the difference in diameters:

arithmetic mean: $\overline{\Delta l} = 49{,}999\,54 \text{ mm}$

standard deviation of a single observation: $s(\Delta l) = 0{,}33\,\mu\text{m}$

standard deviation of the mean: $s(\overline{\Delta l}) = \dfrac{s(\Delta l)}{\sqrt{5}} = 0{,}15\,\mu\text{m}$

The standard deviation of a single observation $s(\Delta l) = 0{,}18\,\mu\text{m}$ takes into account effects due to form deviations of the ring to be calibrated as well as due to the repeatability of the comparator. To obtain the standard uncertainty of measurement to be associated with the observed mean difference of the diameters, the uncertainty resulting from the zeroing of the comparator display must also be taken into account. This is deduced from the pooled estimate of the standard deviation $s_\text{p}(0) = 0{,}25\,\mu\text{m}$ obtained in a prior measurement under the same conditions of measurement. The resulting standard measurement uncertainty to be associated with the observed diameter difference is:

$$u(\Delta l) = \sqrt{s^2(\overline{\Delta l}) + s_\text{p}^2(0)} = 0{,}30\ \mu\text{m}$$

S13.10 Uncertainty budget (d_X)

quantity X_i	estimate x_i	standard uncertainty $u(x_i)$	probability distribution	sensitivity coefficient c_i	uncertainty contribution $u_i(y)$
d_S	40,000 7 mm	0,10 μm	normal	1,0	0,10 μm
Δl	49,999 55 mm	0,30 μm	normal	1,0	0,30 μm
δl_i	0,0 mm	0,22 μm	rectangular	1,0	0,22 μm
δl_T	0,0 mm	0,15 μm	normal	1,0	0,15 μm
δl_P	0,000 004 mm	0,0065 μm	rectangular	1,0	0,0065 μm
δl_E	0,0 mm	0,018 μm	rectangular	1,0	0,018 μm
δl_A	0,0 mm	0,012 μm	rectangular	1,0	0,012 μm
d_X	90,000 25 mm				0,433 μm

S13.11 Expanded uncertainty

$$U = k \cdot u(d_X) = 2 \cdot 0{,}433\,\mu\text{m} \cong 0{,}9\,\mu\text{m}$$

S13.12 Reported result

The diameter of the ring gauge is (90,000 3 ± 0,000 9) mm.

The reported expanded uncertainty of measurement is stated as the standard uncertainty of measurement multiplied by the coverage factor $k = 2$ which for a normal distribution corresponds to a coverage probability of approximately 95%.

S13.13 Mathematical note on non-coaxiality

Since it is not possible to make an exact adjustment of the rings with respect to the measuring axis of the comparator, the quantity determined in the measurement is a chord of the respective ring in the proximity of its diameter. The length d' of this chord, which is observed in the measurement, is related to the diameter of the ring d by

$$d' = d \cdot \cos(\delta\varphi) \cong d \cdot \left(1 - \frac{1}{2}(\delta\varphi)^2\right) \qquad \text{(S13.6)}$$

where $\delta\varphi$ is the small angle that complements half of the central angle of the chord to $\pi/2$. This angle is related on the other hand to the small distance δc of the chord form the centre of the ring by

$$\delta c = \frac{1}{2} \cdot d \cdot \sin(\delta\varphi) \cong \frac{1}{2} \cdot d \cdot \delta\varphi \qquad \text{(S13.7)}$$

so that eq. (S13.6) may be rewritten as

$$d' \cong d - 2\frac{(\delta c)^2}{D} \tag{S13.8}$$

where the diameter d of the ring in the ratio has been replaced by its nominal diameter D since the nominator of the ratio is a small quantity already. The best estimate of the diameter is obtained by taking the expectation of the last relation to be

$$d = d' + 2\frac{u^2(\delta c)}{D} \ . \tag{S13.9}$$

Here it has been taken into account that the small distance δc has zero expectation. It must also be kept in mind that the meaning of d, d' and δc in eq. (S13.8) and eq. (S13.9) is not identical; whereas in eq. (S13.8) these symbols represent the not-exactly known quantities or random variables, in eq. (S13.9) they stand for the expectations of these quantities. Since the variance of a random variable equals the expectation of the square of its deviation from the respective expectation, the square of the standard measurement uncertainty to be associated with the diameter of the ring is, according to eq. (S13.8),

$$u^2(d) = u^2(d') + 4\cdot(\alpha - 1)\frac{u^4(\delta c)}{D^2} \tag{S13.10}$$

with

$$\alpha = \frac{m_4(\delta c)}{m_2^2(\delta c)} \tag{S13.11}$$

being the ratio of the 4th order centred moment to the square of its 2nd order centred moment of the small distance δc. This ratio depends on the distribution that is assumed for δc. It takes the value $\alpha = 9/5$ if δc is assumed to be rectangularly distributed so that in this case the standard measurement uncertainty to be associated with the diameter is expressed by

$$u^2(d) = u^2(d') + \frac{16}{5}\cdot\frac{u^4(\delta c)}{D^2} \tag{S13.12}$$